A Road Trafic Speed-Accidents Optimization Model

Author
Daniel Hailemariam Kassa
Dept of Mathematics, College of Natural and Computational Sciences,
Adigrat University, Adigrat, Ethiopia
Email: *danielhailish2123@gmail.com or danielhailish212327@gmail.com*

Abstract
Studies summarizing findings from before and after studies of the impacts of speed on accidents have resulted in a rule of thumb saying that a 1 km/h decrease in mean speed causes a 3 percent reduction in injury accidents.
Currently the crash risk and road risk are protected by speed limit. According to international context the Speed limits are set as less than 50km/hr in urban. This is because of factors such as high crash risk and high road traffic accidents. However, only speed limit cannot minimize the crash risk and road traffic accident.
In this model, the objective function is a road traffic speed- accidents risk itself and each constraint are the sum of speed of vehicles per each partition and per each round.
By using this optimization model, we can calculate the road accidents risk of every vehicle. As a result these risks will be converted to means of income by introducing tax to each vehicle base on their speed.
The purpose of this optimization model is to minimize the road accidents risk and to maximize government income.

Introduction

Human Life is a valuable asset for a country. Accidents and Medical emergencies such as fire, road accidents, heart attacks etc occur every day. It is critical for emergency teams to reach the accident spot so as to save human lives. Thus, hospitals and fire stations are spread evenly across the city to reduce response time. However rapid population growth in cities and rural has resulted in high traffic densities on city roads and rural roads. This is an additional hindrance to fire, ambulance and other emergency vehicles.

According to WHO (2009), however, the increase in road transportation has placed a considerable burden on the people's lives. The fatality of road traffic deaths and injuries is the major one. According to Ethiopian Federal Police (2008/09-2010/11) report, each year more than two thousand people die and ten thousand people are injured because of road traffic accidents. WHO (2009) asserted that road traffic crashes are the leading causes of death and disablement of people under 44 years next to HIV/AIDS.

The American review (Transportation Research Board 1998) summarizes what is known about the relationship between speed and accidents in the following terms:

"Drivers' speed choices impose risks that affect both the probability and severity of crashes. Speed is directly related to injury severity in a crash. The probability of severe injury increases sharply with the impact speed of a vehicle in a collision, reflecting the laws of physics. Although injury to vehicle occupants in a crash can be mitigated by safety belts and airbags, the strength of the relationship between speed and crash severity alone is sufficient reason for managing speed."

According to Finch, D. J., Kompfner, P., Lockwood, C. R. & Maycock, G. (1994)

1 km/h increase in speed implies 3% increase in accidents. Much study shows the higher the speed, the steeper the increase in accident risk.

Based on the rule of thumb, the road traffic speed-accidents optimization model will be as follows:

The General formulation of Minimum road traffic speed- accidents risk (in urban):

$$Min \; T(x_1, x_2, \dots x_n)$$
$$Subject \; to \; S\,(v_1, v_2, \dots, v_n) \geq 50n$$

Where $T(x_1, x_2, \dots x_n)$ is road traffic speed-accidents risk function with each *road traffic – speed accidents risk* $x_1, x_2, \dots x_n$ of a vehicles and $S(v_1, v_2, \dots, v_n)$ is a speed functions of each vehicles corresponding to each x_i and n is the number of rounds for each vehicles in a road.

**The Decentralized system can be formulated as:**

$$Min \; \sum_{i=1}^{n} x_i$$
$$Subject \; to: \; v_1 + v_2 + \cdots + v_n \geq 50n$$
$$a_{11}v_1 + a_{12}v_2 + \cdots + a_{1n}v_n \geq 50n$$
$$\vdots$$
$$a_{m1}v_1 + a_{m2}v_2 + \cdots + a_{mn}v_n \geq 50n$$

Where, by a rule of thumb $x_i = (1 + a_{ij})0.03v_i$ is the road accidents risk and a_{ij} , $1 \leq j \leq m$ and

$1 \leq i \leq n$ is a scalar. . Since the time t_i per each length l_i is $t_i = \frac{l_i}{v_i}$, then we have a relation

$$x_i = (1 + a_{ij})0.03 \frac{l_i}{t_i}.$$

For Example: (1) consider the transportation of minibus from urban A to urban B.
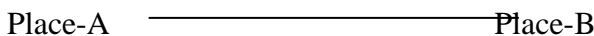
Place-A ——————————— Place-B

From the above figure,

Let Mr.X, drive his minibus for the purpose income from a place-A to a place-B.

Suppose he was finishing the first round with speeds v_1 & v_2 in two partitions, he was also finished the second round with speeds $2v_1$ & v_2.

Lastly, at the third round, he was also finished with speed v_1 & $3v_2$. Then find the solutions v_1 & v_2 and the minimum road accidents risk.

Solution:

The above problem can tabulated as:

Table-1:

Rounds	Speeds(km/hr)	
R-1	V_1	V_2
R-2	$2V_1$	V_2
R-3	V_1	$3V_2$

Based on the above formulated model, the problem can be formulated as:

Min: $Z = 0.12v_1 + 0.15v_2$

S.t $v_1 + v_2 \geq 100$
        $2v_1 + v_2 \geq 100$
        $v_1 + 3v_2 \geq 100$

Where $v_1, v_2 \geq 0$
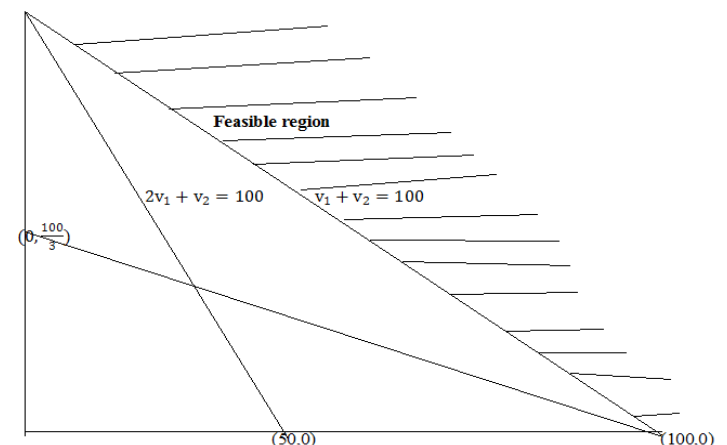
Applying Graphical Method:

Consider $v_1 + v_2 = 100$

At $v_1 = 0$, $(v_1, v_2) = (0,100)$ and At $v_2 = 0$, $(v_1, v_2) = (100,0)$

Consider $2v_1 + v_2 = 100$

At $v_1 = 0$, $(v_1, v_2) = (0,100)$ and At $v_2 = 0$, $(v_1, v_2) = (50,0)$

Again, consider $v_1 + 3v_2 = 100$

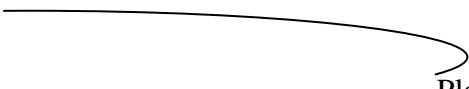At $v_1 = 0$, $(v_1, v_2) = (0, \frac{100}{3})$ and At $v_2 = 0$, $(v_1, v_2) = (100,0)$

To minimize $Z = 0.12v_1 + 0.15v_2$

At $(v_1, v_2) = (0, \frac{100}{3}) \Rightarrow Z = 5$, at $(v_1, v_2) = (100,0)$, $\Rightarrow Z = 12$ *and* at $(v_1, v_2) = (0,100) \Rightarrow Z = 15$

Therefore, the unbounded solution is at (100,0) and $Z_{min} = 12$

Hence, Mr.X contribute 12 minimum road accidents risk from place -A to place-B.

Example :(2) consider the figure below:

Place-C

Place-D

Let Mr.Z drive his taxi for the purpose of income from place-C to place-D as shown in the figure above. The speeds he took for each round is given as the table-2 below:

Table-2:

Rounds	Speeds(km/hr)	
R-1	V_1	V_2
R-2	$1.5V_1$	V_2
R-3	$1.2V_1$	$1.3V_2$

Then, calculate the minimum road accidents risk.

Solution:

Based on the above formulated model, the problem can be formulated as:

Min: $Z = 0.111v_1 + 0.099v_2$

S.t $v_1 + v_2 \geq 100$

   $1.5v_1 + v_2 \geq 100$

   $1.2v_1 + 1.3v_2 \geq 100$

Where $v_1, v_2 \geq 0$

Clearly $v_1 =$ 1.46E-7 and $v_2 = 100$ and$Z_{min} = 9.9$.

Hence, Mr.Z contribute 9.9 minimum road accidents risk from place -C to place-D

Conclusions

There are different reasons why road accidents happen. The main reason is almost all the vehicles use a maximum speed in and outsides of the town. Most studies show that 30-50% of a road accidents happened because of speed.

But according to the traffics rules there is a restrictions of speed in both in and out sides of the urban.

Frankly to speak, some drivers derive their vehicles using a proper speed for the purpose of their work and some of them are not.

Currently the crash risk and road risk are protected by speed limit. However, only speed limit cannot minimize the crash risk and road traffic accident.

From the above two problem and the formulated model, Mr.X contribute 12 minimum road accidents risk and Mr. Z also contribute 9.9 minimum road accidents risk.

So, if the government introduce tax to each vehicles based on their speed, then this formulated model will have multi objective. Such as minimize road accident risk, maximizing government's income and minimize CO_2.

That means, if the government introduce tax to speed of vehicles, the above road accidents risk made by Mr. X and Mr. Z will be converted to means of income in the second case Mr. X and Mr. Z will drive their vehicles properly and carefully. As a result the road accidents risk will be minimized.

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