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Effect of Gravitation on the Motion of Cylindrical Imploding Shock Wave in Dusty Gas

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Abstract:

In the present paper, the propagation of cylindrical imploding strong shock waves in a self-gravitating dusty gas has been investigated by Chester- Chisnell- Whitham method. The dusty gas is assumed to be a mixture of small solid spherical particles and a perfect gas. The effect of overtaking disturbances on freely propagating shock has been estimated by using Yadav technique [Mod. Mes. Cont. B, 46(4), 1(1992)]. Assuming an initial density distribution $\rho_o = \mu \log(\varphi r)$, where μ is the density at the axis of symmetry and φ is a constant, the variation of shock velocity, shock strength, pressure and particle velocity with propagation distance, density parameter (φ), mass concentration of solid particles in the mixture and the ratio of solid particles to the initial density of gas have been computed and discussed through figures. The effect of self-gravitation is also calculated. Maintaining equilibrium flow conditions, it is found that the presence of dust particles in the gases medium has significant effects on the variation of flow variables and the shock is strengthened under the influence of overtaking disturbances. The role of the acceleration due to gravity on propagation of shock wave is also calculated numerically and depict through graphs. Finally, the results obtained here have been compared with those for pure perfect gas as well as those for freely propagating shock.

Keywords: Shock wave propagation, strong, imploding, dusty gas, CCW method.

1 Introduction

The study of shock wave propagation in a gas containing solid particles is of immense importance in astrophysical phenomenon especially in the star formation, where gravitational force governs the phenomena to the large extent. Many authors have investigated the motion of shock wave in dusty gaseous medium (Sedov[1], Marble[2], Pai et al[3], Miura and Glass[4], Gretler and Regenfelder [5], Hirschler and Steiner [6]). Propagation of shock waves in a dusty gas with exponentially varying density has been considered by Ray and Bhowmick[7], Vishwakarma *et. al.* [8] and Viashwakarma[9] using similarity method. He found that the presence of small solid particles in the medium has significant effect on the variation of density and pressure. Neglecting the effect of overtaking disturbances, Yadev *et. al.*[10] investigated the behavior of weak cylindrical shock waves in a mixture of perfect gas and small solid particles in presence of magnetic field. Vishwakarma and Nath[11] found similarity solutions for an unsteady isothermal and adiabatic flow behind a strong exponential shock driven out by the piston in a dusty gas. The effects of variation of radiation parameter and time on the motion of shock wave propagation in dusty gas with radiation heat flux has been studied by Singh and Vishwakarm[12]. The effect of overtaking disturbances on the propagation of shock wave has been studied by Yousaf ([13], [14]) using similarity solutions. Effect of overtaking disturbances on the motion of shock wave in an self-gravitating ideal gas has been found by Yadav ([15], [16]) on including the effect of overtaking disturbances. The problem of adiabatic propagation of strong spherical converging shock in an ideal gas has been tackled by Yadav and Gangwar[17] including effect of overtaking disturbances.

In present study, the propagation of cylindrical converging strong shock waves in a non-uniform dusty ideal gas has been investigated by Chester[18]- Chisnell[19]-Whitham[20] method. The effect of overtaking disturbances on the freely propagation of shock is included using Yadav method[21]. The non-uniformity arises due to the self-gravitation of the medium and also from the motion of the flow behind the propagating shock. It is assumed that the dusty gas is a mixture of a perfect gas and large number of small spherical solid particles of uniform size, which are uniformly distributed in the gas. For simplicity of calculations, the initial value of initial volume fraction of solid particles in the mixture is a small constant. The small solid particles behave like a pseudo-fluid and the equilibrium flow condition is maintained in the flow field. It is assumed that the viscous stress and heat conduction of the mixture are negligible. We also assumed that the particles do not interact on each other and there thermal motion is negligible.

Assuming an initial density distribution law as $\rho_o=\mu\log(\varphi r)$, where μ is the density at the axis of symmetry and φ is a constant, the variation of shock velocity, shock strength, pressure and particle velocity with propagation distance(r), density parameter (φ), mass concentration of solid particles in the mixture(K_p) and the ratio of solid particles to the initial density of gas(σ) have been computed and discussed through figures simultaneously for freely propagation and under the influence of overtaking disturbances. The effect of self-gravitation on all flow variables have been shown through graphs. It is found that the presence of dust particles in the gaseous medium has significant effects on the variation of flow variables and the shock is strengthened under the influence of overtaking disturbances.

Finally, the results obtained here have been compared with those for perfect gas [15] as well as with those for freely propagating shock.

2 Theory

2.2 Basic Equation

The equations governing the cylindrically symmetrical flow of a mixture of gas and small spherical solid particles enclosed by the shock front under the influence of its own gravitation are

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{1}{\rho}\frac{\partial p}{\partial r}+g=0 \qquad (1)$$

$$\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial r}+\rho\frac{\partial u}{\partial r}+\frac{\rho u}{r}=0 \qquad (2)$$

$$\frac{\partial \varepsilon}{\partial t}+u\frac{\partial \varepsilon}{\partial r}+\frac{p}{\rho^2}\left(\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial r}\right)=0 \qquad (3)$$

where g, u, p, ρ, and ε denote respectively, the acceleration due to gravity of earth, the flow velocity, the pressure and density at a distance $'r'$ from the origin at time $'t'$, and ε is the internal energy of the mixture.

The equation of state of the mixture of perfect gas and small solid particles is given by [3]

$$p=\rho RT\frac{(1-k_p)}{1-Z} \qquad (4)$$

where R is the gas constant, T the temperature, Z is the volume fraction of solid particles in the mixture, K_p the mass concentration of solid particles in the mixture.

The relation between k_p and Z is as follows

$$k_p=\frac{Z\rho_{sp}}{\rho} \qquad (5)$$

where ρ_{sp} is the species density of solid particles. In equilibrium flow, K_p is a constant in the whole flow field.

The internal energy of the mixture can be written as follows[3]

$$U_m = \left[K_p C_{sp} + \left(1-K_p\right)C_v\right]T = C_{vm}T \qquad (6)$$

where C_{sp} is the specific heat of solid particles, C_v the specific heat of the gas at constant volume process and C_{vm} the specific heat of the mixture at constant volume process.

The specific heat of the mixture at constant pressure process is

$$C_{pm} = K_p C_{sp} + \left(1-K_p\right)C_p \qquad (7)$$

where C_p is the specific heat of the gas at constant pressure process.

The ratio of specific heat of the mixture is given by [2] and [3]

$$\Gamma = \frac{C_{pm}}{C_{vm}} = \frac{\gamma + \delta\Phi}{1+\delta\Phi} \qquad (8)$$

where $\Gamma = \frac{C_p}{C_v}$, $\delta = \frac{K_p}{1-K_p}$ and $\Phi = \frac{C_{sp}}{C_v}$, (9)

The internal energy is therefore, given by

$$\varepsilon = \frac{p(1-Z)}{\rho(\Gamma-1)} \qquad (10)$$

The jump conditions across the strong shock are given by [9]

$$u = (1-\beta)U \qquad (11)$$

$$\rho = \rho_o/\beta \qquad (12)$$

$$p = (1-\beta)\rho_o U^2 \qquad (13)$$

$$Z = Z_o/\beta \qquad (14)$$

$$a = \sqrt{\frac{\Gamma p}{\rho(1-Z)}} = \xi U \qquad (15)$$

and $a_o = \sqrt{\frac{\Gamma p_o}{\rho_o(1-Z_o)}}$ (16)

where $\beta = \frac{\Gamma - 1 + 2Z_o}{\Gamma+1}$, $\xi = \sqrt{\frac{\Gamma\beta^2(1-\beta)}{(\beta - Z_o)}}$ (17)

and U=dR/dt denotes the shock velocity, R is the shock radius, the suffix "$_o$" refers to the values in front of the shock., $U/a_o=M$ is the Mach number, 'a' the speed of sound in the equilibrium 2-phase flow and β is the shock density ratio which is an unknown parameter to be determined.

The initial volume fraction of the solid particles Z_o is, in general not a constant, but the volume occupied by the solid particles is very small because the density of the solid particles is much larger than that of the gas [22], hence Z_o may be assumed a small constant and is given by[23]

$$Z_o = \frac{K_p}{\sigma\left(1-k_p\right)+k_p} \qquad (18)$$

where σ is the ratio of solid particles to the gas(specific density or density ratio)

Assuming the medium in which shock propagates is at rest and at uniform pressure, but initial density distribution

$$\rho_o = \mu \log(\varphi r) \qquad (19)$$

where, μ is the density at the axis of symmetry and φ is a constant.

On substituting $u = 0 = \partial/\partial t$, $p = p_o$ and $\rho = \rho_o$ in equation (1), the condition of hydrostatic equilibrium prevailing in front of the shock is written as

$$\frac{1}{\rho_o}\frac{dp_o}{dr} + g = 0 \qquad (20)$$

Substituting the value of ρ_o from (19) in equation (20), we get

$$dp_o = g\mu \log(\varphi r)dr \qquad (21)$$

On integrating equation (21), we get

$$p_o = g\mu r\left[1-\log(\varphi r)\right] \qquad (22)$$

and $a_o = \sqrt{\frac{gr\Gamma\{1-\log(\varphi r)\}}{(1-Z_o)\log(\varphi r)}}$ (23)

(a) Freely propagation of shock

For cylindrical imploding shocks, the characteristic form of the system of equations (1)-(4), i.e. the form in which each equation contain derivatives in only one direction in (r, t) plane, is

$$dp - \rho a du + \frac{\rho a^2 u}{(u-a)}\frac{dr}{r} - \frac{\rho a}{(u-a)} g\, dr = 0 \qquad (24)$$

Substituting the values from equations (11)-(16) in equation (24), we have

$$dU = \frac{1}{(2\beta-\xi)}\left[\begin{array}{c}\dfrac{\xi g}{(1-\beta)(1-\beta-\xi)U} \\ -\dfrac{\beta U}{r\log(\varphi r)} - \dfrac{\xi^2 U}{(1-\beta-\xi)r}\end{array}\right] dr \qquad (25)$$

This equation is used to compute the strength of the cylindrical shock propagating freely (FP).

(b) Effect of overtaking disturbances

For C_- disturbances, using equation (25), equation (11) gives

$$du_- = \frac{1-\beta}{(2\beta-\xi)}\left[\begin{array}{c}\dfrac{\xi g}{(1-\beta)(1-\beta-\xi)U} \\ -\dfrac{\beta U}{r\log(\varphi r)} - \dfrac{\xi^2 U}{(1-\beta-\xi)r}\end{array}\right] dr \qquad (26)$$

To consider the effect of overtaking disturbances, independent C_+ characteristic is considered. The differential equation valid across C_+ disturbances is written as

$$dp + \rho a du + \frac{\rho a^2 u}{(u+a)}\frac{dr}{r} + \frac{\rho a}{(u+a)} g dr = 0 \qquad (27)$$

Substituting jump conditions (11-16) in equation (27), we have

$$dU + \frac{1}{(2\beta+\xi)U}\left[\begin{array}{c}\dfrac{\beta U^2}{r\log(\varphi r)} + \dfrac{\xi^2 U^2}{(1-\beta-\xi)r} \\ + \dfrac{\xi g}{(1-\beta)(1-\beta-\xi)}\end{array}\right] dr = 0 \qquad (28)$$

For C_+ disturbances, above equation gives

$$du_+ = -\frac{1-\beta}{(2\beta+\xi)}\left[\begin{array}{c}\dfrac{\xi g}{(1-\beta)(1-\beta-\xi)U} \\ +\dfrac{\beta U}{r\log(\varphi r)} + \dfrac{\xi^2 U}{(1-\beta-\xi)r}\end{array}\right] dr \qquad (29)$$

In presence of both C_- and C_+ disturbances, the fluid velocity behind the imploding shock is given by[21]

$$du_- - du_+ = (1-\beta)\, dU$$

(30)

Substituting values from equations (26) and (29), in equation (30), we have

$$dU^* = \left[\frac{1}{(2\beta-\xi)}\left\{\begin{array}{c}\dfrac{\xi g}{(1-\beta)(1-\beta-\xi)U} \\ -\dfrac{\beta U}{r\log(\varphi r)} - \dfrac{\xi^2 U}{(1-\beta-\xi)r}\end{array}\right\} + \frac{1}{(2\beta+\xi)}\left\{\begin{array}{c}\dfrac{\xi g}{(1-\beta)(1-\beta-\xi)U} \\ +\dfrac{\beta U}{\log(\varphi r)} + \dfrac{\xi^2 U}{(1-\beta-\xi)}\end{array}\right\}\right] dr \qquad (31)$$

where '*' represents the respective values modified under the effect of overtaking disturbances.

This equation is used to compute the strength of the cylindrical shock under the influence of overtaking disturbances (EOD).

3. Results and Discussions

We consider that shock is strong. For strong shock we initially take

$\gamma = 1.4,\ k_p = 0.4,\ \Phi = 1,\ \mu = 1.28,$

$\varphi = 0.2,\ g = 9.8,\ \pi = 3.14$ and $\sigma = 50$

The equation (31) and (23) is used to prepare profile of shock strength of adiabatic strong shock propagating freely in non-uniform dusty atmosphere. The non-uniformity of atmosphere arises due to its self-gravitation and also the flow behind the shock propagates. It shows that shock strength (U/a_o) is a function of propagation distance (*r*), concentration of mass particle (k_p), specific density or density ratio(σ), adiabatic index (γ), acceleration due to gravity (g) and density parameter (φ). When effect of overtaking disturbances is taken into account, the expression modifies to equation (31).

The variation of shock strength (U/a_o), shock velocity (U), particle velocity (u/a_o) and non-dimensional pressure(p/p_o) with propagation distance (*r*) for γ=1.4 is obtained by numerically integrating equations (25) and (31) by Runge-Kutta method is presented in Figures(1-4) for freely propagation and also under the effect of overtaking disturbances. It is found that shock strength increases with propagation distance for freely propagation as well as with effect of overtaking disturbances. It is pronounced here that shock is strengthened with overtaking disturbances. It is also found that strength of overtaking wave disturbances increases as shock implodes in the mixture. The dotted line in the Figures(1-4) represents the values of respective variables for the case of non-dusty gaseous atmosphere. The similar variation has been obtained by [10]. The strength of strong cylindrical shock wave decreases as concentration of solid particles (k_p) increases in the medium.

The variation of shock strength, shock velocity, particle velocity and non-dimensional pressure with density ratio (σ), density parameter (φ) and acceleration due to gravity (g) is obtained for r=14 and are depicted through figures (5-16), respectively, for freely propagation and under the influence of overtaking disturbances. It is observed that shock strength increases for all the cases.

It is clear from the Figures(4-8) that all flow variables and their modified values decrease as the specific density(σ) of the dusty medium increases. As an increase in density parameter(φ), the velocity of cylindrical shock is decreases[cf. Figure (9)] whereas all other variables are increases[cf. Figure (10-12)]. It is observed from the figures(13-16) as acceleration due to gravity is increases shock strength(U/a_o), particle velocity(u/a_o) and pressur(p/p_o) is decreases while a negligible change have been found in shock velocity(U).

Finally, it is concluded from this study that effect of self-gravitation plays a significant role in the propagation of shock wave in dusty atmosphere.

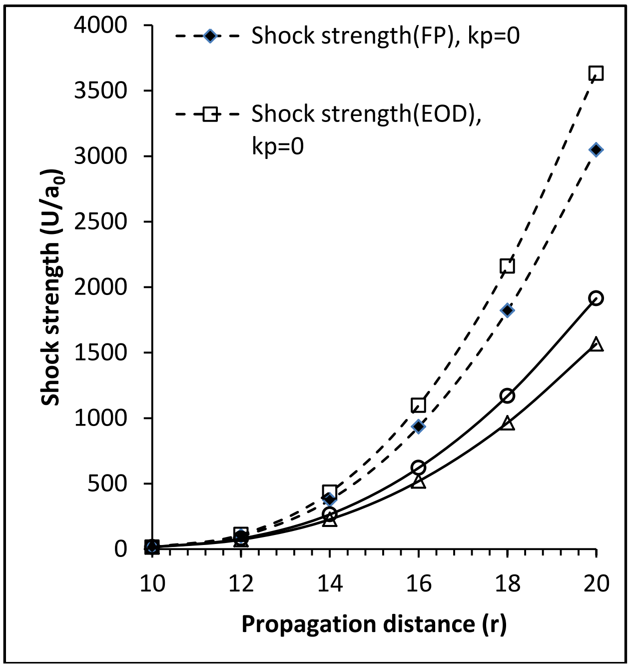

Figure 1: Variation of shock strength (U/a_o) with propagation distance(r).

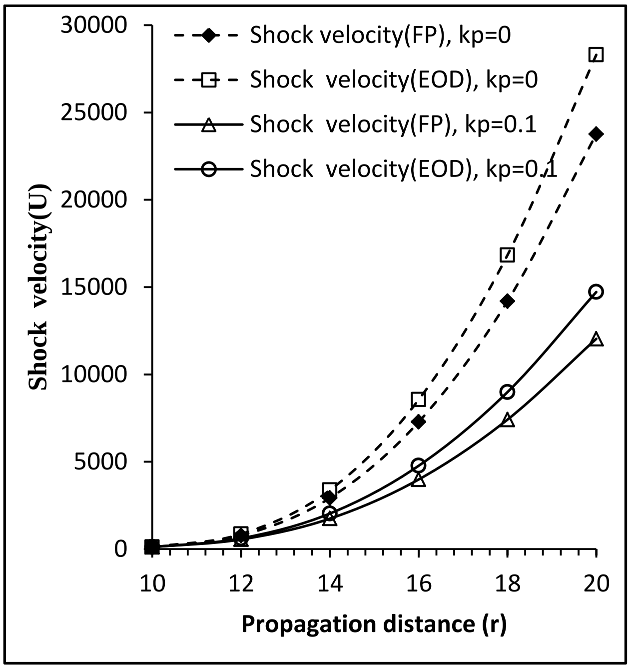

Figure 2: Variation of shock velocity(U) with propagation distance(r).

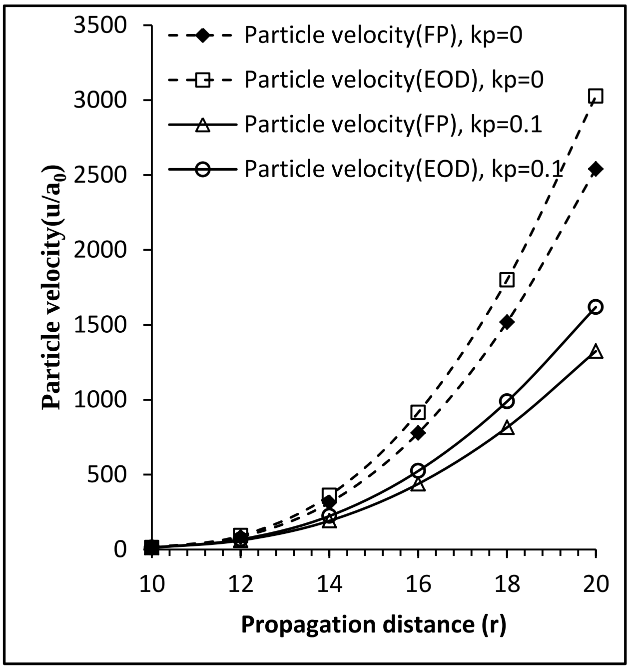

Figure 3: Variation of particle velocity(u/a_0) with propagation distance(r).

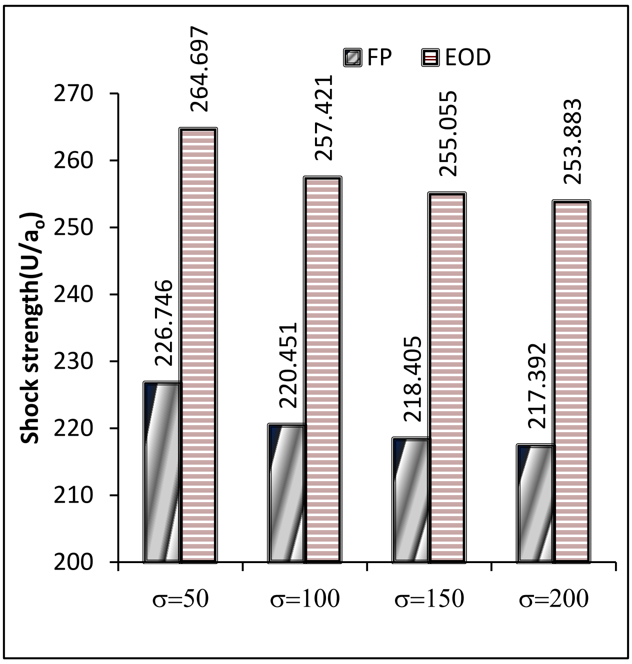

Figure 5: Variation of shock strength (U/a_0) specific density (σ) at r=14.

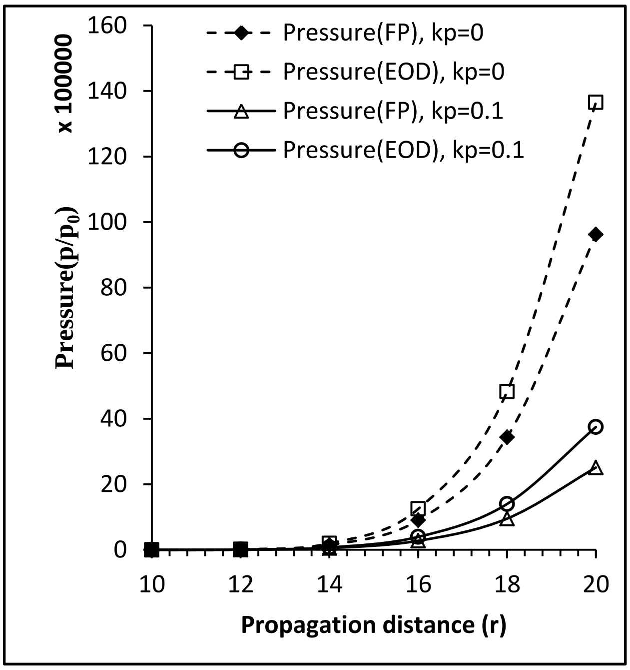

Figure 4: Variation of pressure (p) with propagation distance(r).

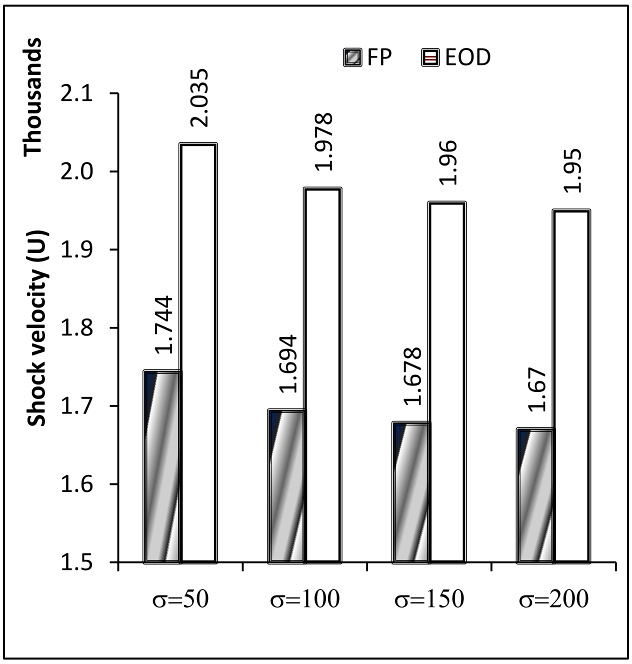

Figure 6: Variation of shock velocity(U) with specific density (σ) at r=14.

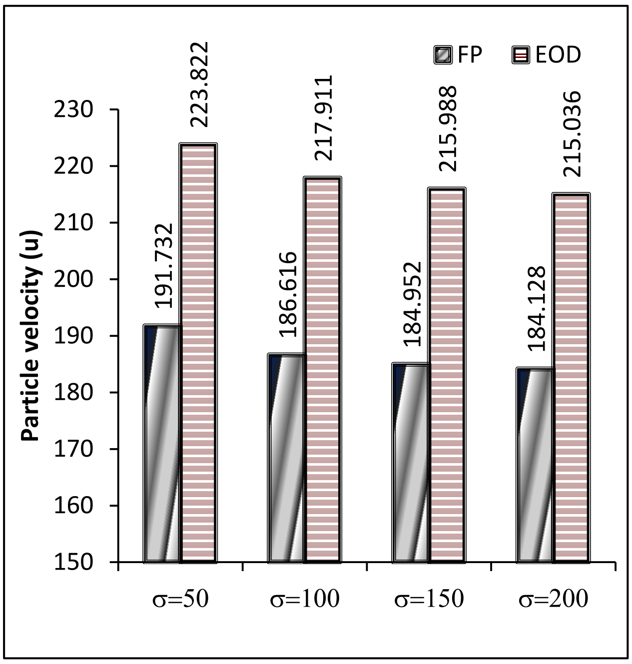

Figure 7: Variation of particle velocity(u/a_0) specific density (σ) at r=14.

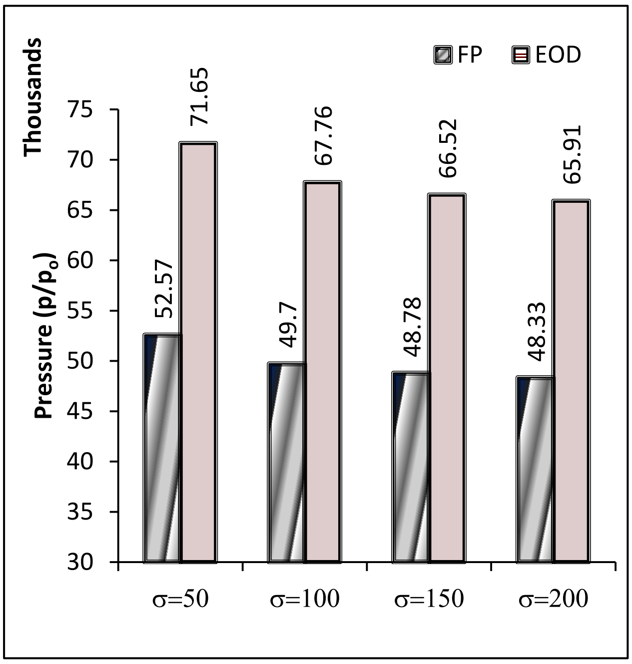

Figure 8: Variation of pressure (p/p_0) specific density (σ) at r=14.

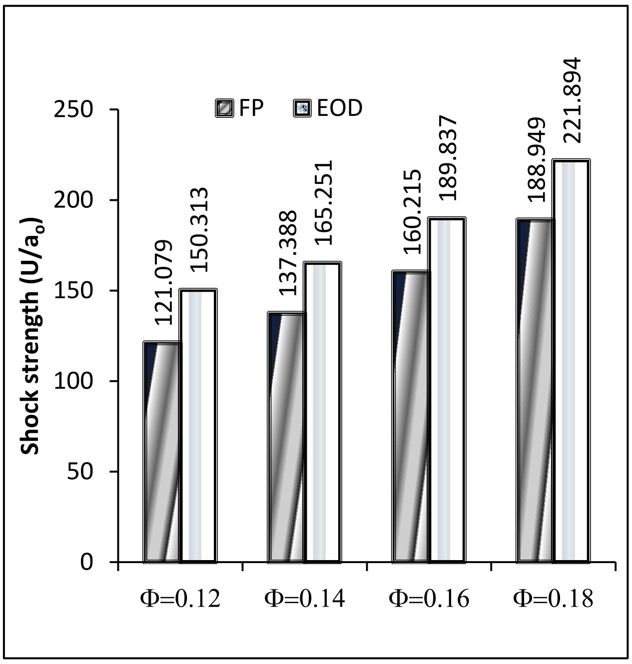

Figure 9: Variation of shock strength (U/a_0) with density parameter(φ).

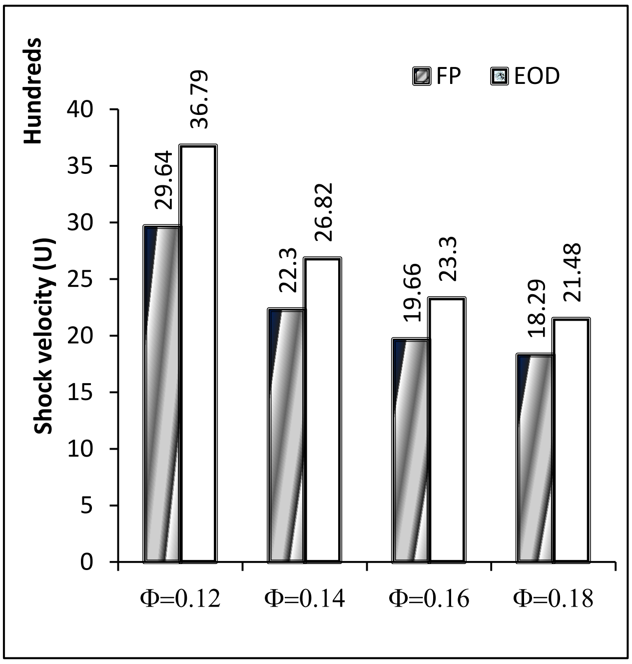

Figure 10: Variation of shock velocity(U) with density parameter(φ).

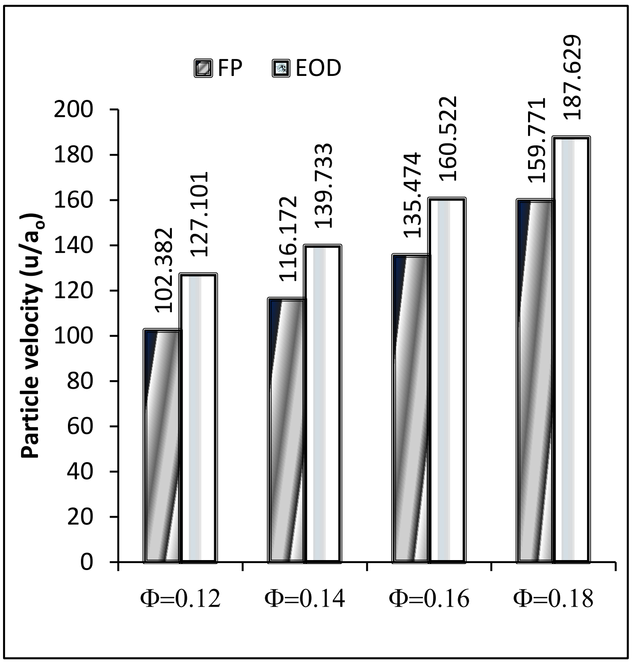

Figure 11: Variation of particle velocity(u/a_0) with density parameter(φ)

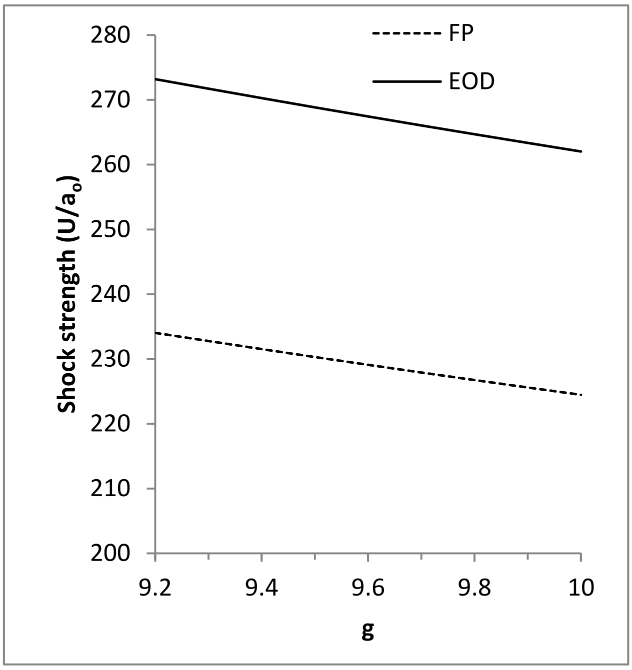

Figure 13: Variation of shock strength (U/a_0) with g.

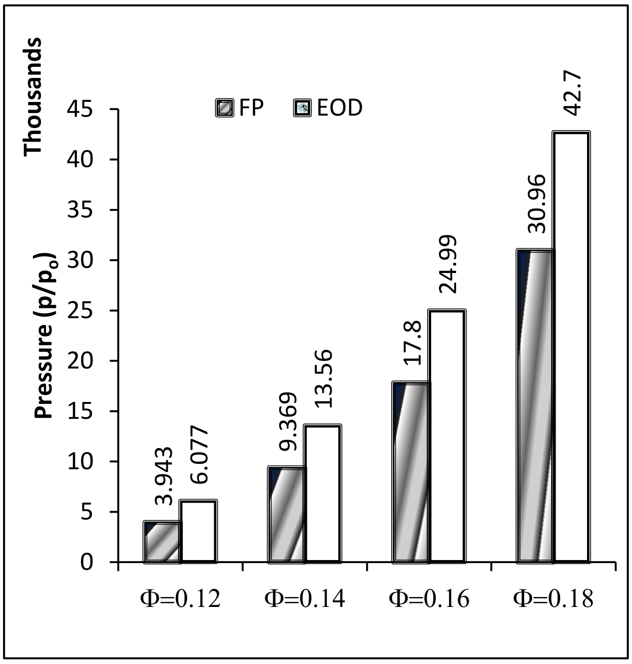

Figure 12: Variation of pressure (p/p_0) with density parameter(φ).

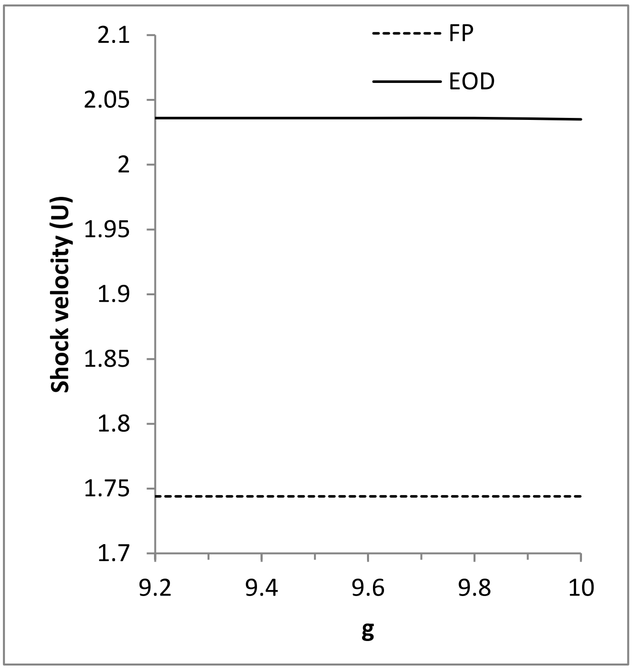

Figure 14: Variation of shock velocity(U) with g.

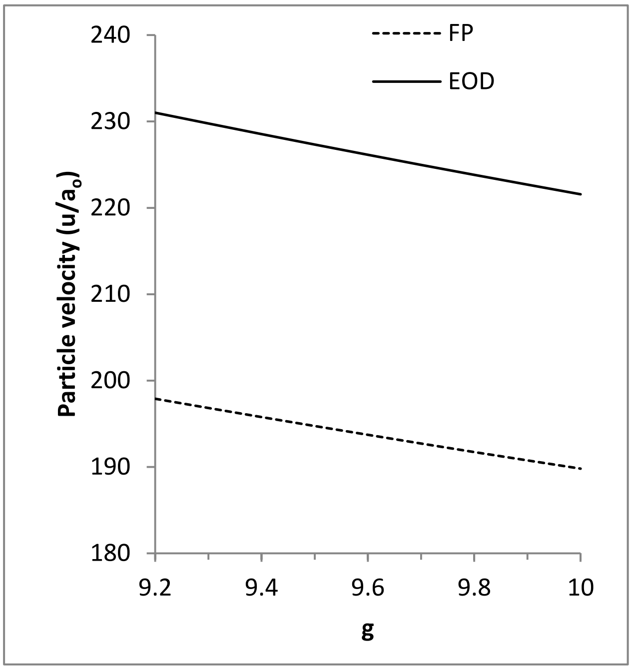

Figure 15: Variation of particle velocity(u/a$_0$) with g.

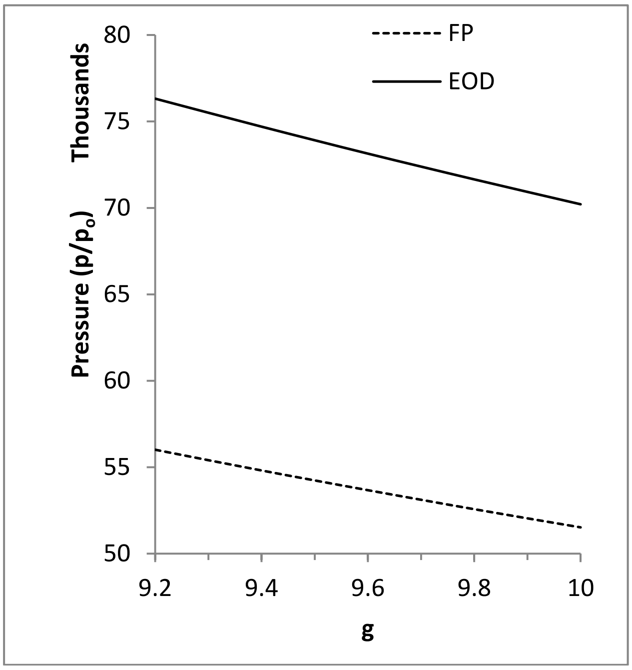

Figure 16: Variation of pressure (p/p$_0$) with g.

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