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Study on Strongly Pseudo Irregular Fuzzy Graphs

Authors
S.P.Nandhini[1], Dr.M.Kamaraj[2]
[1]Department of Mathematics
The Standard Fireworks Rajaratnam College For Women, Sivakasi-626 123
Tamilnadu, India
[2]Department of Mathematics
Government Arts and Science College, Sivakasi.
Tamilnadu, India

Abstract: -In this paper, Some results on Strongly pseudo Irregular Fuzzy Graphs and Strongly pseudo total Irregular Fuzzy Graphs are established. Comparative study between Strongly pseudo Irregular Fuzzy Graphs and Strongly pseudo total Irregular Fuzzy Graphs is done. Also defined Δ- pseudo domination set and Δ-pseudo total dominating set in irregular fuzzy graph.
Keywords:- 2-degree, pseudo degree of a vertex in a graph, irregular fuzzy graph, totally irregular fuzzy graph, Δ-pseudo dominating set and Δ-pseudo total dominating set.

1.Introduction

In this paper , we consider only finite, simple, connected graphs. We denote the vertex set and the edge set of a graph G by V(G) and E(G) respectively. The degree of a vertex v is the number of edges incident at v and it is denoted by d(v). A graph G is regular if all its vertices have the same degree. The 2-degree of v is the sum of the degrees of the vertices adjacent to v and it is denoted by t(v). A pseudo degree of a vertex v is denoted by $d_a(v)$ and defined as $\frac{t(v)}{d^*{}_G(v)}$ where $d^*{}_G(v)$ is the number of edges incident at v.

A graph is called pseudo –regular if every vertex of G has equal pseudo (average) degree.

1.1.Review of Literature

NagoorGani and Radha[2] introduced regular fuzzy graphs, total degree and totally regular fuzzy graphs. NagoorGani and Latha[3] introduced neighbourly irregular fuzzy graphs, neighbourly total irregular fuzzy graphs, highly irregular fuzzy graphs and highly total irregular fuzzy graphs. SP.Nandhini and E.Nandhini [4] introduced strongly irregular fuzzy graphs, strongly total irregular fuzzy graphs. N.R.S.Maheswari and C.Sekar[6] introduced pseudo degree and total pseudo degree in fuzzy graphs and pseudo regular fuzzy graphs and discussed some of its properties . N.R.S.Maheswari and M.Sudha[7] introduced pseudo irregular fuzzy graphs and highly pseudo irregular fuzzy graphs and discussed some of its properties .N.R.S.Maheswari and M.Rajeswari [5] introduced strongly pseudo ir regular fuzzy graphs discussed some of its properties .These motivates us to compare strongly pseudo irregular and strongly pseudo total irregular fuzzy graphs . Also some results on strongly pseudo irregular fuzzy graphs and strongly pseudo total irregular fuzzy graphs are studied.

Throughout this paper only undirected fuzzy graphs are considered.

2.Preliminaries:-

Definition 2.1: A fuzzy graph G = (σ, μ) is a pair of functions σ: V → [0, 1] and μ: V x V → [0, 1],where for all u, v ∈V, we have μ(u, v) ≤ σ(u) Λ σ(v). A fuzzy graph G is called complete fuzzy graph if the relation μ(u, v) =σ(u) Λ σ(v).

Definition 2.2 : A partial fuzzy graph $\xi' = (V, \tau, \rho)$ of ξ is such that $\tau(v) \le \sigma(v)$ for all $v \in V$, and $\rho(u, v) \le \mu(u, v)$ for all $u, v \in V$. Fuzzy graph $\xi' = (P, \sigma', \mu')$ of ξ is such that $P \subset V$, $\sigma'(v) \le \sigma(v)$ for all $u \in P$ and $\mu'(u, v) \le \mu(u, v)$ for all $u, v \in P$.

Definition 2.3: The underlying crisp graph of a fuzzy graph $G = (\sigma, \mu)$ is denoted by $G^* = (\sigma^*, \mu^*)$, where $\sigma^* = \{u \in V / \sigma(u) > 0\}$ and $\mu^* = \{ (u, v) \in V \times V / \mu(u, v) > 0\}$.

Definition 2.4: Let $G = (\sigma, \mu)$ be a fuzzy graph.
The degree of a vertex u is $d_G(u) = d(u) = \sum_{u \neq v} \mu(u, v) = \sum_{uv \in E} \mu(u, v)$.

Definition 2.5: Let $G = (\sigma, \mu)$ be a fuzzy graph on G*. The total degree of a Vertex $u \in V$ is defined by $td_G(u) = \sum_{u \neq v} \mu(u, v) + \sigma(u) = \sum_{uv \in E} \mu(u, v) + \sigma(u) = d_G(u) + \sigma(u)$.

Definition2.6: Let $G = (\sigma, \mu)$ be a fuzzy graph. Then G is irregular, if there is a vertex which is adjacent to vertices with distinct degrees.

Definition 2.7: Let $G = (\sigma, \mu)$ be a fuzzy graph such that $G^* = (V, E)$ is a cycle.Then G is a fuzzy cycle if and only if there does not exist a unique edge (x, y) such that
$\mu(x, y) = \Lambda \{\mu(u, v) / (u, v) > 0\}$.

Definition 2.8: Let $G = (\sigma, \mu)$ be a fuzzy graph. Then G is totally irregular, if there is a vertex which is adjacent to vertices with distinct total degrees.

Definition 2.9[5]: Let $G = (\sigma, \mu)$ be a fuzzy graph. The 2-degree of a vertex is defined as the sum of degrees of the vertices incident at v and it is denoted by t(v). i.e $t(v) = \sum_{uv \in E} d_G(u)$, where $d_G(u)$ is the degree of the vertex u in fuzzy graph G which is adjacent to the vertex v

A pseudo degree of a vertex v is defined as $d_a(u) = \dfrac{t(v)}{d_{G^*}(v)}$ where $d_{G^*}(v)$ is the number of edges incident at v in underlying graph.

Definition 2.10[6]: Let $G = (\sigma, \mu)$ be a connected fuzzy graph. On G* (V, E) . The total pseudo degree of vertex v in G is denoted by $td_a(u) = d_a(v) + \sigma(v)$ for all $v \in V$.

Definition 2.11: Let $G = (\sigma, \mu)$ be a connected fuzzy graph. G is said to be a neighbourly pseudo irregular fuzzy graph if every two adjacent vertices of G have distinct pseudo degree.

Definition 2.12: If every two adjacent vertices of a fuzzy graph $G = (\sigma, \mu)$ have distinct pseudo total degree, then G is said to be a neighbourly pseudo total irregular fuzzy graph.

Definition 2.13[7]: Let $G = (\sigma, \mu)$ be a connected fuzzy graph. G is said to be a highly pseudo irregular fuzzy graph if every vertex of G is adjacent to vertices with distinct pseudo degrees.

Definition 2.14[7]: Let $G = (\sigma, \mu)$ be a connected fuzzy graph. G is said to be a highly pseudo total irregular fuzzy graph if every vertex of G is adjacent to vertices with distinct pseudo total degrees.

Definition 2.15[5]: Let $G = (\sigma, \mu)$ be a connected fuzzy graph. G is said to be a strongly pseudo irregular fuzzy graph if every pair of vertices in G have distinct pseudo degrees.

Definition 2.16[5]: Let G = (σ, μ) be a connected fuzzy graph. G is said to be a strongly pseudo total irregular fuzzy graph if every pair of vertex in G have distinct pseudo total degrees.

3.Strongly Pseudo Irregular Fuzzy Graphs

Theorem 3.1:

A Fuzzy graph $G = (\sigma, \mu)$ where G^* is a cycle with vertices 3 is Strongly Pseudo irregular iff the weights of the edges between every pair of vertices are all distinct.

Proof: Let *u*,*v*,& *w* are the vertices of *G*. For if the weights of any two edges *uv* & *vw* are the same, *i.e.*$\mu(u, v) = \mu(v, w)$

$\Rightarrow .\mu(u, v) + \mu(u, w) = \mu(v, w) + \mu(u, w)$

$\Rightarrow d(u) = d(w)$

$\Rightarrow d(u) + d(v) = d(w) + d(v)$

$\Rightarrow \frac{t(w)}{2} = \frac{t(u)}{2}$

$\Rightarrow \frac{t(w)}{d_G^*(w)} = \frac{t(u)}{d_G^*(u)}$

$\Rightarrow d_a(w) = d_a(u)$ which is contradicts the definition of Strongly Pseudo irregular fuzzy graph.

Conversely

Weights of edges between every pair of vertices are all distinct.

Suppose $d_a(u) = d_a(v)$

$\Rightarrow \frac{t(u)}{d_G^*(u)} = \frac{t(v)}{d_G^*(v)}$

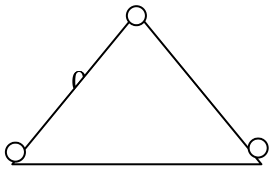
$\Rightarrow \frac{t(u)}{2} = \frac{t(v)}{2}$

$\Rightarrow t(u) = t(v)$

$\Rightarrow d(v) + d(w) = d(u) + d(w)$

$\Rightarrow \mu(v, w) + \mu(u, v) = \mu(u, v) + \mu(u, w)$ *which is a*$\Rightarrow\Leftarrow$

Example 3.2:

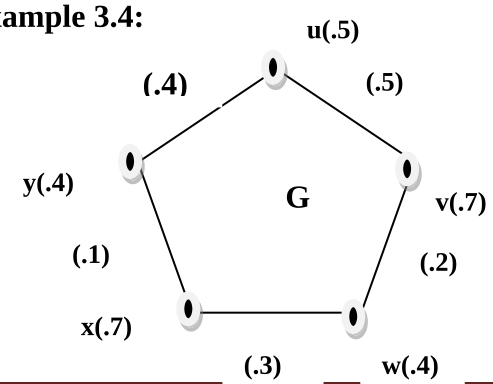


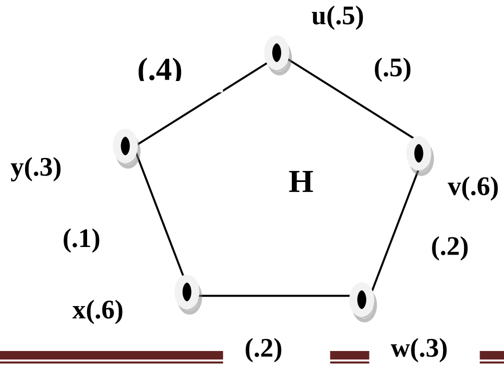
$$d_a(u) = 1.5 \neq d_a(v) = 1.3 \neq d_a(w) = 1.4$$

Theorem 3.3: A Fuzzy graph $G = (\sigma, \mu)$ is Strongly Pseudo irregular then the Partial fuzzy Subgraph $H = (\tau, \rho)$ of G need not be a Strongly Pseudo Irregular fuzzy graph.

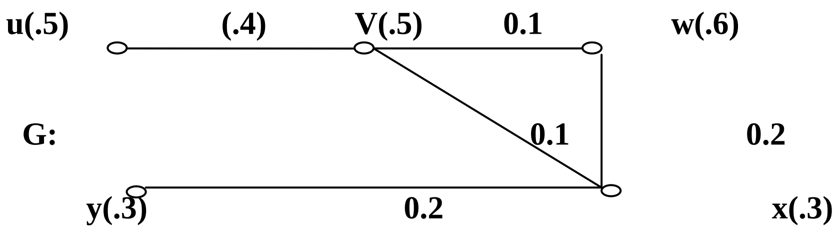
Proof: To every vertex the adjacent vertices with distinct degrees or the non-adjacent vertices with distinct degrees may happen to be the vertices with same degree is *H*.

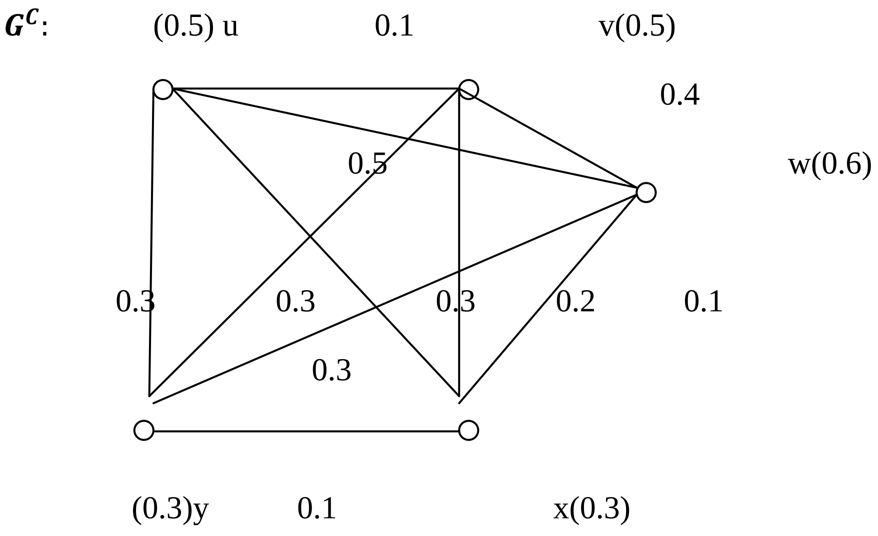
Example 3.4:





Theorem 3.5: Cycle C_n with $n=4$ *is not* Strongly Pseudo irregular Fuzzy graph.
Proof: Suppose the vertices of C_4 are v_1, v_2, v_3, & v_4.
Let $v_1\, e_1\, v_2\, e_2\, v_3\, e_3\, v_4\, e_4\, v_1$ forms a closed walk.
The non-adjacent vertices v_1 &v_3 have the same adjacent vertices.
∴ v_1 & v_3 have the same 2- degree. Hence v_1 &v_3 have the same Pseudo degree.
Similarly v_1 &v_4 also have the same Pseudo degree.
Theorem 3.6: Let $G = (\sigma, \mu)$ be a highly pseudo irregular Fuzzy and neighbourly pseudo Fuzzy graph $G = (\sigma, \mu)$. If every pair of vertices in G is either adjacent or incident on the same vertex then G is strongly pseudo irregular Fuzzy graph.
Proof: Let $G = (\sigma, \mu)$ be a fuzzy graph. Suppose every pair of vertices in G is either adjacent or incident on the same vertex.
Since $G = (\sigma, \mu)$ is both highly pseudo and neighbourly pseudo irregular fuzzy graph, every vertices of G have distinct pseudo degrees.
∴$G = (\sigma, \mu)$ is strongly Pseudo irregular fuzzy graph.
Theorem 3.7: If $G = (\sigma, \mu)$ is a Highly pseudo and neighbourly pseudo irregular fuzzy graph then G need not be a Strongly pseudo irregular Fuzzy graph.
Proof: The vertices u & v of G, which are not adjacent & not incident on the same vertex may happen to have same pseudo degrees.
This contradicts the definition of strongly pseudo irregular.
Theorem 3.8: The Complement of a strongly pseudo irregular Fuzzy graph need not be Strongly Pseudo irregular.
Proof: To every vertex, the adjacent vertices with distinct pseudo degrees or the non-adjacent vertices with distinct Pseudo degrees may happen to be adjacent vertices with same pseudo degrees or non adjacent vertices with same pseudo degrees. This contradicts the definition of strongly pseudo irregular Fuzzy graph.





d(u)=1.2 d(v)=1 d(w)=1.3 d(x)=0.7 d(y)=1

$d_a(u) = \frac{1+1.3+0.7+1}{4} = 1$ $\quad d_a(v) = \frac{1.2+1.3+0.7+1}{4} = 1.5$ $\quad d_a(w) = \frac{1.2+1+0.7+1}{4} = 0.97$

$d_a(x) = \frac{1.2+1+1.3+1}{4} = 1.12$ $\quad d_a(y) = \frac{1.2+1+1.3+0.7}{4} = 1.5$

Here $d_a(v) = d_a(y)$

Theorem 3.9: Let $G = (\sigma, \mu)$ be a Fuzzy graph, where G^* is regular, σ is a constant function and $\mu(u,v)<\sigma(u)\wedge\sigma(v)$ $\forall\ u,v \in V(G)$.Then G is a strongly pseudo irregular Fuzzy graph iff G^C is a strongly pseudo irregular Fuzzy graph.

Proof: Let $G = (\sigma, \mu)$ be a Strongly pseudo irregular Fuzzy graph & $\sigma(u)=C\ \forall\ u \in G$.

$\Leftrightarrow d_a(u) \neq d_a(v)$

$\Leftrightarrow \frac{t(u)}{d_G^*(u)} \neq \frac{t(v)}{d_G^*(v)}$

$\Leftrightarrow \frac{\sum d(x_i)}{d_G^*(u)} \neq \frac{\sum d(y_j)}{d_G^*(v)}$

Where x_i are the vertices incident on u & y_j are the vertices incident on v.

Since G is regular,

$\Leftrightarrow \frac{\sum[\sum \mu(x_i,u')]}{d_G^*(u)} \neq \frac{\sum[\sum \mu(y_j,v')]}{d_G^*(v)}$ Where u' are the vertices incident on x_i & v' are the vertices incident on y_j.

$\Leftrightarrow \frac{\sum[\sum(C - \mu(x_i,u'))]}{d_G^*(u)} \neq \frac{\sum[\sum(C - \mu(y_j,v'))]}{d_G^*(v)}$

$\Leftrightarrow \frac{\sum[\sum(\sigma(x_i) \wedge \sigma(u) - \mu(u,x_i)]}{d_G^*(u)} \neq \frac{\sum[\sum(\sigma(y_j) \wedge \sigma(v) - \mu(v,y_j)]}{d_G^*(v)}$

$\Leftrightarrow \frac{\sum[\sum \mu^c(u,x_i)]}{d_G^*(u)} \neq \frac{\sum[\sum \mu^c(v,y_j)]}{d_G^*(v)}$

$\Leftrightarrow \frac{\sum d(x_i)}{d_G^*(u)} = \frac{\sum d(y_j)}{d_G^*(v)}$

Where x_i are the vertices adjacent to u in G^C & y_j are the vertices adjacent to v in G^C.

$\Leftrightarrow t(u) = t(v)$

$\Leftrightarrow \frac{t(u)}{d_G^*(u)} = \frac{t(v)}{d_G^*(v)}$ (Since G^* is regular then $(G^C)^*$ is also regular)

$\Leftrightarrow d_a(u) = d_a(v)\ \forall u,v \in G^C$

$\therefore G^C = (\sigma^c, \mu^c)$ is Strongly Pseudo irregular fuzzy graph.

Theorem 3.10: Let $G = (\sigma, \mu)$ be a Complete Fuzzy graph. If G is a Strongly Pseudo irregular Fuzzy graph then G^C is not a Strongly irregular Pseudo Fuzzy graph.

Proof: Let the Complete Fuzzy graph $G = (\sigma, \mu)$ is Strongly Pseudo irregular Fuzzy graph.

Then $\mu(u)\wedge\mu(v)=\mu(u,v)\ \forall uv\in E$

$\Rightarrow \mu(u)\wedge\mu(v)-\mu(u,v)=0$

$\Rightarrow \mu^c(u,v) = 0\ \ \forall u,v \in G^C$

$\Rightarrow d(u) = 0\ \forall u \in G^C$

$\Rightarrow \sum_{uv\in E} d_G(u) = 0$

$\Rightarrow t(u) = 0\ \ \forall u \in G^C$

$\Rightarrow d_a(u) = 0\ \forall u \in G^C$

$\therefore G^C$ is not a Strongly Pseudo irregular Fuzzy graph.

Theorem 3.11: Let $G = (\sigma, \mu)$ be a fuzzy graph on $G^*(V,E)$.If G is neighbourly pseudo irregular and highly pseudo irregular Fuzzy graph then the pseudo degrees of all vertices of G need not be distinct.

Proof: Let G is neighbourly pseudo & highly pseudo irregular fuzzy graph

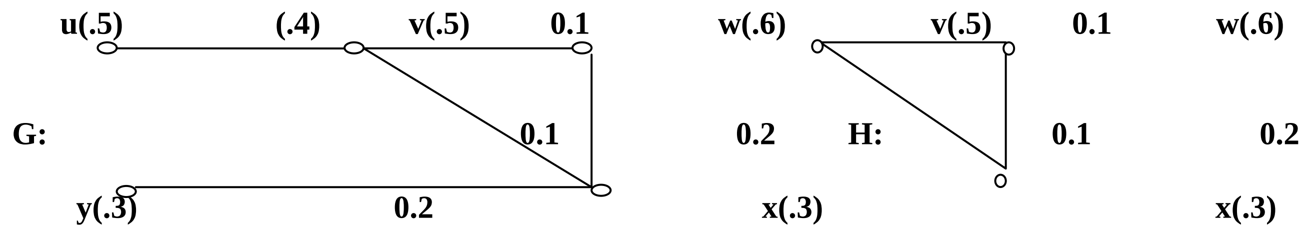
Let *u* and *v* are vertices which are not adjacent and not incident on same vertex. Then *u* and *v* may happen to have same pseudo degree without affecting neighbourly pseudo irregularity and highly pseudo irregularity.

∴The pseudo degrees of all vertices of *G* need not be distinct.

Theorem 3.12: The fuzzy subgraph $H = (\tau, \rho)$ of a strongly Pseudo irregular Fuzzy graph $G = (\sigma, \mu)$ need not be Strongly Pseudo irregular.

Proof: To every vertex the adjacent vertices with distinct degrees or the non-adjacent vertices with distinct degrees may happen to be the vertices with same pseudo degree in *H*.

Example 3.13:



In H, $d_a(w) = d_a(x) = 0.25$

Theorem 3.14: The underlying crisp graph G* = (σ*,μ*)of a fuzzy graph G = (σ, μ) is complete then G is a neighbourly Pseudo irregular fuzzy graph if and only if G is a strongly Pseudo irregular fuzzy graph .

Proof: Let the underlying crisp graph G* = (σ*,μ*)of a fuzzy graph G = (σ, μ) is complete. Then Every two vertices are adjacent.

Suppose G is neighbourly pseudo irregular fuzzy graph.

⇔every two adjacent vertices have distinct pseudo degrees.

⇔every vertices of G have distinct pseudo degrees.

⇔G is a strongly pseudo irregular fuzzy graph.

Theorem 3.15: The underlying crisp graph G* = (σ*,μ*) of a fuzzy graph G = (σ, μ) is complete then G is a highly Pseudo irregular if and only if G is a strongly pseudo irregular. **Proof:** Let the underlying crisp graph G* = (σ*,μ*)of a fuzzy graph G = (σ, μ) is complete with n vertices. Every vertex of G is adjacent to remaining (n-1) vertices.

⇔every vertex of G is adjacent to vertices with distinct pseudo degrees.

⇔every vertices of G have distinct pseudo degrees.

⇔G is a strongly pseudo irregular fuzzy graph.

4.Strongly pseudo totally irregular Fuzzy graph

Theorem 4.1:A fuzzy graph $G = (\sigma, \mu)$ where G* is a cycle with vertices 3 and σ is a constant function then G is strongly pseudo total irregular if and only if the weights of the edges between every pair of vertices are all distinct.

Proof: Let *u*,*v*,& *w* are the vertices of *G*. For if the weights of any two edges *uv* & *vw* are the same, *i.e.*$\mu(u, v) = \mu(v, w)$

$\Rightarrow \mu(u, v) + \mu(u, w) = \mu(v, w) + \mu(u, w)$

$\Rightarrow d(u) = d(w)$

$\Rightarrow d(u) + d(v) = d(w) + d(v) \Rightarrow \quad t(w) = t(u)$

$\Rightarrow \frac{t(w)}{2} = \frac{t(u)}{2}$

$\Rightarrow d_a(w) + \sigma(v) = d_a(u) + \sigma(u) \Rightarrow \; td_a(u) = td_a(v)$which is contradicts the definition of Strongly Pseudo total irregular fuzzy graph.

Conversely

Weights of edges between every pair of vertices are all distinct.---------- (1)

Suppose $td_a(u) = td_a(v)$

$\Rightarrow d_a(u) + \sigma(u) = d_a(v) + \sigma(v) \quad \Rightarrow d_a(u) = d_a(v)$

$\Rightarrow \frac{t(u)}{2} = \frac{t(v)}{2}$

$\Rightarrow t(u) = t(v)$
$\Rightarrow d(v) + d(w) = d(u) + d(w)$
$\Rightarrow .\mu(v,w) + \mu(u,v) = \mu(u,v) + \mu(u,w) \Rightarrow \mu(u,v) = \mu(v,w)$
which is $\Rightarrow\Leftarrow$ to(1)
$\therefore$ *G* is Strongly Pseudo total irregular.

Theorem 4.2: Let $G = (\sigma, \mu)$ highly pseudo total irregular and neighbourly pseudo total irregular fuzzy graph .If every pair of vertices in *G* is either adjacent or incident on the same vertex then *G* is strongly pseudo total irregular.

Proof: Suppose every pair of vertices is either adjacent or incident on the same vertex.
Since $G = (\sigma, \mu)$ is both highly pseudo total irregular and neighbourly pseudo total fuzzy graph, every vertices have distinct pseudo total degrees.
Therefore G is strongly pseudo total irregular fuzzy graph.

Theorem 4.3: A highly pseudo total irregular and neighbourly pseudo total irregular Fuzzy graph $G = (\sigma, \mu)$ need not be a Strongly pseudo total irregular fuzzy graph

Proof: Suppose *u* and *v* be any two vertices of *G*, which are not adjacent and not incident on the same vertex may happen to have same pseudo total degrees. This contradicts the definition of Strongly pseudo total irregular fuzzy graph.

Theorem 4.4: Let $G = (\sigma, \mu)$be a fuzzy graph ,where G^* is regular, σ is a constant function and $\mu(u, v)<\mu(u) \wedge \mu(v)$ for all $u,v \in V(G)$.Then G is a strongly pseudo total irregular fuzzy graph iff G^C is a strongly pseudo total irregular fuzzy graph.

Proof: Let *G= (σ, μ)* be a strongly Pseudo irregular fuzzy graph and *σ(u) =c for all* $u \in G$ *and G* is a K-regular graph with n vertices*

$td_a(u) \neq td_a(v)\ \forall\ u, v \epsilon V(G)$
$\Rightarrow \sigma(u) + d_a(u) \neq \sigma(v) + d_a(v)$

$$\Rightarrow \sigma(u) + \left[\frac{t\ (u)}{k}\right] \neq \sigma(v) + \left[\frac{t\ (v)}{k}\right]$$

$$\Rightarrow \sigma(u) + \left[\frac{\sum d(x_i)}{k}\right] \neq \sigma(v) + \left[\frac{\sum d(y_j)}{k}\right]$$

$$\Rightarrow \sigma(u) + \frac{1}{k}\sum\left[\sum_{s=1}^{k} \mu(x_{i,}u_s)\right] \neq \sigma(v) + \frac{1}{k}\sum\left[\sum_{S=1}^{k} \mu(y_{j,}v_s)\right] \text{ for all } u_s \text{ adjacent to } x_i$$

for all v_s adjacent to y_j

$$\Rightarrow \frac{1}{k}\sum\left[\sum_{s=1}^{n-1} (c - \mu(x_{i,}u_s))\right] \neq +\frac{1}{k}\sum\left[\sum_{s=1}^{n-1} (c - \mu(y_{j,}v_s))\right]$$

$$\Rightarrow \sigma(u) + \frac{1}{n-1}\sum\left[\sum_{s=1}^{n-1} \mu^c(x_{i,}u\ )\right] \neq \sigma(v) + \frac{1}{n-1}\sum\left[\sum_{s=1}^{n-1} \mu^c(y_{j,}v)\right]$$

$$\Rightarrow \sigma(u) + \frac{1}{n-1}\sum d(x_i) \neq \sigma(v) + \frac{1}{n-1}\sum d(y_j), x_i \in G^c, y_j \in G^c$$

$$\Rightarrow \sigma(u) + \left[\frac{t\ (u)}{n-1}\right] \neq \sigma(v) + \left[\frac{t\ (v)}{n-1}\right] \forall u, v \in G^c$$

$\Rightarrow td_a(u) \neq td_a(v)\ \forall\ u, v \epsilon G^c$

Theorem 4.5: Let *G= (σ, μ)* be a Complete Fuzzy graph. If *G* is a Strongly pseudo total irregular fuzzy graph with σ is constant then G^c is not a Strongly pseudo total irregular Fuzzy graph.

Proof: Let the Complete Fuzzy graph *G= (σ, μ)* is Strongly pseudo irregular Fuzzy graph. By hypothesis σ(u)=k for all u

Then $\mu(u) \wedge \mu(v) = \mu(u,v)\ \forall u,v \in G$
$\Rightarrow \mu(u) \wedge \mu(v) - \mu(u,v) = 0$
$\Rightarrow \mu^c(u,v) = 0\ \ \forall u, v \in G^C$
$\Rightarrow d(u) = 0\ \forall u \in G^C$
$\Rightarrow t(u) = 0\ \ \forall u \in G^C\ \Rightarrow \frac{t(u)}{d_G^*(u)} = 0$

$\Rightarrow d_a(u) = 0\ \forall u \in G^C \Rightarrow \sigma(u) + d_a(u) = \sigma(u) \Rightarrow td_a(u) = k\ \forall u \in G^C$

$\therefore G^C$ is not a Strongly Pseudo total irregular Fuzzy graph.

Theorem 4.6: Let $H = (V, \tau, \rho)$ be a partial Fuzzy subgraph of a Fuzzy graph $G = (V, \sigma, \mu)$ where G^* is regular .If τ is constant function, $\tau(u) < \sigma(u)$ and $\rho(u, v) = \mu(u, v) \forall\ u, v \in V$ then G is a Strongly pseudo irregular iff H is a Strongly pseudo total irregular Fuzzy graph.

Proof: Let $\sigma(u, v)$ be a Strongly pseudo irregular Fuzzy graph.

Since $\rho(u, v) = \mu(u, v) \forall\ u, v \in V$,

pseudo degree of u_i in G = pseudo degree of u_i in H.

$\therefore d_a\ (u_i)$ in G=$d_a\ (u_i)$ in H--- *(2)* $\forall u_i \in V$

$G = (\sigma, \mu)$)be a Strongly pseudo irregular Fuzzy graph

$\Leftrightarrow d_a(u_i) \neq d_a(u_j) \forall u_i \in V(G) i \neq j$

$\Leftrightarrow d_a(u_i) \neq d_a(u_j) \forall u_i \in V(H) i \neq j$ by (2)

$\Leftrightarrow \tau(u) + d_a(u_i) \neq \tau(u) + d_a(u_j) \forall u_i \in V(H) i \neq j$

$\Leftrightarrow td_a(u_i) \neq td_a(u_j) \forall u_i \in V(H) i \neq j$

$\Leftrightarrow H = (\tau, \rho)$ is a Strongly pseudo total irregular Fuzzy graph.

Theorem 4.7: Let $H = (V, \tau, \rho)$ be a partial Fuzzy Subgraph of Fuzzy graph $G = (V, \sigma, \mu)$ where G^* is regular. If $\tau\ \&\ \sigma$ are constant function , $\tau <\ \sigma$ and $\rho(u, v) = \mu(u, v) \forall\ u, v \epsilon V$ then G is a Strongly pseudo total irregular fuzzy graph iff H is a Strongly pseudo total irregular fuzzy graph.

Proof: Since $\rho(u, v) = \mu(u, v) \forall\ u, v \in V$,

Pseudo degree of u_i in G = Pseudo degree of u_i in H.

$\therefore d_a\ (u_i)$ in G=$d_a\ (u_i)$ in H--- *(3)* $\forall u_i \in V$

If G is a Strongly Pseudo total irregular fuzzy graph.

$\Leftrightarrow td_a(u_i) \neq td_a(u_j) \forall u_i \in V(G) i \neq j$

$\Leftrightarrow \sigma(u_i) + d_a(u_i) \neq \sigma(u_j) + d_a(u_j) \forall u_i, u_j \in V(G) i \neq j$

$\Leftrightarrow d_a(u_i) \neq d_a(u_j) \forall u_i, u_j \in V(G) i \neq j$

$\Leftrightarrow d_\alpha(u_i) \neq d_\alpha(u_j) \forall u_i, u_j \in V(H) i \neq j$ (by (3))

$\Leftrightarrow \sigma(u_i) + d_\alpha(u_i) \neq \sigma(u_j) + d_a(u_j) \forall u_i, u_j \in V(H) i \neq j$

$\Leftrightarrow td_a(u_i) \neq td_a(u_j) \forall u_i \in V(H) i \neq j$

$\Leftrightarrow H$ is a Strongly Pseudo total irregular Fuzzy graph.

Theorem 4.8: The underlying crisp graph G* = (σ*,μ*)of a fuzzy graph G = (σ, μ) is complete then G is a neighbourly pseudo total irregular fuzzy graph if and only if G is a strongly pseudo total irregular fuzzy graph .

Proof: Let the underlying crisp graph G* = (σ*,μ*)of a fuzzy graph G = (σ, μ) is complete. Then Every two vertices are adjacent.

Suppose G is neighbourly pseudo total irregular fuzzy graph.

⇔every two adjacent vertices have distinct Pseudo total degrees.

⇔every vertices of G have distinct Pseudo total degrees.

⇔G is a strongly pseudo total irregular fuzzy graph.

Theorem 4.9: The underlying crisp graph G* = (σ*,μ*) of a fuzzy graph G = (σ, μ) is complete then G is a highly pseudo total irregular if and only if G is a strongly pseudo total irregular Fuzzy graph.

Proof: Let the underlying crisp graph G* = (σ*,μ*)of a fuzzy graph G = (σ, μ) is complete with n vertices. Every vertex of G is adjacent to remaining (n-1) vertices.

Suppose G is highly pseudo total irregular Fuzzy graph.

⇔every vertex of G is adjacent to vertices with distinct pseudo total degrees.

⇔every vertices of G have distinct pseudo total degrees.

⇔G is a strongly pseudo total irregular fuzzy graph.

Theorem 4.10: If σ is a constant function, Cycle C_n with n=4 is not Strongly pseudo total irregular Fuzzy graph.

Proof: The non-adjacent vertices $v_1\ \&\ v_3$ and $v_2\ \&\ v_4$ have the same pseudo degree.

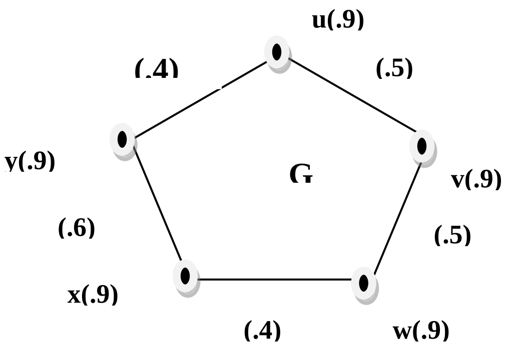
Since σ is a constant function, the non-adjacent vertices v_1 & v_3 and v_2 & v_3 have the same pseudo total degree.
$\therefore C_n$ with n=4 is not Strongly pseudo total irregular Fuzzy graph.

5. Δ −Pseudo Domination in Irregular Fuzzy Graph:

Definition 5.1: A set S $\subseteq$V in a pseudo irregular fuzzy graph G = (σ, μ) is called a Δ −Pseudo dominating set if for every $u \in V\text{-}S$ there exists $v \in S$ such that u and v are adjacent in G and d(v)=maximum Pseudo degree (Δ_{ps}(G)).

Definition 5.2: A set S $\subseteq$V in a pseudo irregular fuzzy graph G = (σ, μ) is called a Δ −Pseudo total dominating set if for every $u \in V\text{-}S$ there exists $v \in S$ such that u and v are adjacent in G and d(v)=maximum Pseudo total degree (Δ_{pst}(G)).

Example 5.3:



Δ-Pseudo dominating set = {u,w} , Δ-pseudo total dominating set ={u,w}

Theorem 5.4: If G is a neighbourly pseudo irregular fuzzy graph and if S is a Δ-pseudo dominating set of G then V- S is not a Δ-pseudo dominating set.

Proof: Let S be a Δ-pseudo dominating set of G. Let u and v are adjacent in G and
$u \in S$, $v \in V\text{-}S$ and $d(u) = \Delta_{ps}(G)$.
Suppose $V\text{-}S$ is a Δ_{ps} dominating set then $d(v) = \Delta_{ps}(G), v \in V\text{-}S$
Which contradicts the definition of neighbourly pseudo irregular.

Theorem 5.5: If G is a neighbourly pseudo total irregular fuzzy graph and if S is a Δ-pseudo total dominating set of G then $V\text{-}S$ is not a Δ-pseudo total dominating set.

Proof: Similar as theorem 5.5

Theorem 5.6: If G is a strongly pseudo irregular fuzzy graph with Δ-pseudo dominating set S then $|S| = 1$.

Proof: Let G is a strongly pseudo irregular fuzzy graph with Δ_{ps}-dominating set S, then no two vertices of G are of same degree.
There exists only one vertex of degree Δ_{ps} say u.
Since G contains S, $u \in S$ dominates all other vertices of G. $\therefore |S| = 1$.

Theorem 5.7: If G is a strongly pseudo total irregular fuzzy graph with Δ-pseudo total dominating set S then $|S| = 1$.

Proof: Simillar as theorem 5.6

Theorem 5.8: If G is a pseudo irregular fuzzy graph if S is a Δ-pseudo dominating set of G with $|S| > 1$ then G is not strongly pseudo irregular.

Proof: Suppose S, Δ-pseudo dominating set of G and $|S| > 1$

Then there exist atleast two vertices in S which dominates all the vertices in V-S and $d(u_i) = \Delta_{ps}$ for all $u_i \in S$.
That is there exist more than one vertices in S whose degree is equal to Δ
Which contradicts the definition of strongly pseudo irregular.

Theorem 5.9: If G is a pseudo total irregular fuzzy graph if S is a Δ-pseudo dominating set of G with $|S| > 1$ then G is not strongly pseudo total irregular.

Proof: Simillar as theorem 5.8

Theorem 5.10: Let G is a strongly pseudo irregular fuzzy graph with *n+1* vertices and $S \subseteq V$ is a Δ-pseudo dominating set then $K_{1,n}$ is a induced subgraph of G*.

Proof: Let G is a strongly pseudo irregular fuzzy graph with n+1 vertices.

S a Δ-pseudo dominating set then $|S| = 1. \Rightarrow |V - S| = n.$

Let $u \in S$, then u dominates all the n vertices of V-S, that is u is adjacent to all the n vertices of V-S

Hence $K_{1,n}$ is the induced subgraph of G*.

Theorem 5.11: Let G is a strongly pseudo total irregular fuzzy graph with *n+1* vertices and $S \subseteq V$ is a Δ-pseudo dominating set then $K_{1,n}$ is a induced subgraph of G*.

Proof: Similar as theorem 5.10

6. Conclusion

Graph theory is an extremely useful tool in solving the combinatorial problems in different areas including geometry, algebra, number theory, topology , operations research and computer science. In this paper we compared strongly pseudo irregular and strongly pseudo total irregular fuzzy graph. The necessary and sufficient conditions for an irregular fuzzy graph to be the strongly pseudo irregular fuzzy graphs have been presented. We have defined Δ-pseudo domination for the irregular fuzzy graphs. Some relations about the defined graphs have been proved.

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