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Oblong Difference Mean Prime Labeling of Some Planar Graphs

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Abstract

The labeling of a graph, we mean assign some integers to the vertices or edges (or both) of the graph. Here the vertices of the graph are labeled with oblong numbers and the edges are labeled with mean of the absolute difference of the end vertex values. Here the greatest common incidence number (*gcin*) of a vertex of degree greater than one is defined as the greatest common divisor of the labels of the incident edges. If the *gcin* of each vertex of degree greater than one is 1, then the graph admits oblong difference mean prime labeling. Here we investigate some planar graphs for oblong difference mean prime labeling.

Keywords- Graph labeling, oblong numbers, prime graphs, prime labeling, planar graphs.

Introduction

In this paper we deal with graphs that are simple, finite and undirected. The symbol V and E denote the vertex set and edge set of a graph G . The graph whose cardinality of the vertex set is called the order of G , denoted by p and the cardinality of the edge set is called the size of the graph G , denoted by q . A graph with p vertices and q edges is called a (p,q) - graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [1],[2],[3] and [4]. Some basic concepts are taken from Frank Harary [1]. In this paper we investigated the oblong difference mean prime labeling of some planar graphs.

Definition: 1.1 Let G be a graph with p vertices and q edges. The the greatest common incidence number (*gcin*) of a vertex of degree greater than or equal to 2, is the greatest common divisor of the labels of the incident edges.

Definition: 1.2 An oblong number is the product of a number with its successor, algebraically it has the form $n(n+1)$.The oblong numbers are 2, 6, 12, 20, --

Main Results

Definition 2.1 Let G be a graph with p vertices and q edges . Define a bijection

$f : V(G) \rightarrow \{2,6,12,20, \dots, p(p+1)\}$ by $f(v_i) = i(i + 1)$, for every i from 1 to p and define a 1-1 mapping $f_{odmpl}^* : E(G) \rightarrow$ set of natural numbers N by

$f_{odmpl}^*(uv) = \left\lfloor \frac{f(u)-f(v)}{2} \right\rfloor$. The induced function f_{odmpl}^* is said to be an oblong difference mean prime labeling, if the *gcin* of each vertex of degree at least 2, is one.

Definition 2.2 A graph which admits oblong difference mean prime labeling is called an oblong difference mean prime graph.

Definition 2.1 Let G be the obtained by joining each vertex of a cycle to a central vertex by edges. G is called wheel graph and is denoted by W_n .

Theorem: 2.1 Wheel graph W_n admits oblong difference mean prime labeling, if $n \equiv 0 \pmod{6}$.

Proof :Let $G = W_n$ and let v_1, v_2, \dots, v_{n+1} are the vertices of G .

Here $|V(G)| = n+1$ and $|E(G)| = 2n$.

Define a function $f : V \rightarrow \{2,6,12, \dots, (n + 2)(n + 1)\}$ by

$$f(v_i) = i(i+1) , i = 1, 2, \dots, n+1.$$

For the vertex labeling f , the induced edge labeling f_{odmpl}^* is defined as follows

$$f_{odmpl}^*(v_i v_{i+1}) = (i+1),$$

$$i = 1, 2, \dots, n$$

$$f_{odmpl}^*(v_1 v_n) = \frac{n^2+n-2}{2}$$

$$f_{odmpl}^*(v_i v_{n+1}) = \frac{n^2+3n+2-i^2-i}{2} ,$$

$$i = 1, 2, \dots, n-1$$

Clearly f_{odmpl}^* is an injection.

gcin of $(v_1) = \text{gcd of } \{ f_{odmpl}^*(v_1 v_2) , f_{odmpl}^*(v_1 v_{n+1}) \}$

$$\begin{aligned}
 &= \gcd \text{ of } \left\{ 2, \frac{n^2+3n}{2} \right\} = 1. \\
 \text{gcin of } (v_{i+1}) &= \gcd \text{ of } \left\{ f_{odmpl}^*(v_i v_{i+1}), \right. \\
 &\quad \left. f_{odmpl}^*(v_{i+1} v_{i+2}) \right\} \\
 &= \gcd \text{ of } \{i+1, i+2\} = 1, \quad i = \\
 &1, 2, \dots, n-1 \\
 \text{gcin of } (v_{n+1}) &= \gcd \text{ of } \left\{ f_{odmpl}^*(v_n v_{n+1}), \right. \\
 &\quad \left. f_{odmpl}^*(v_{n+1} v_{n-1}) \right\} \\
 &= \gcd \text{ of } \{n+1, 2n+1\} \\
 &= \gcd \text{ of } \{(n+1), n\} = 1
 \end{aligned}$$

So, **gcin** of each vertex of degree greater than one is 1.

Hence W_n , admits oblong difference mean prime labeling.

Example 2.1 Oblong difference mean prime labeling of wheel graph W_6 .

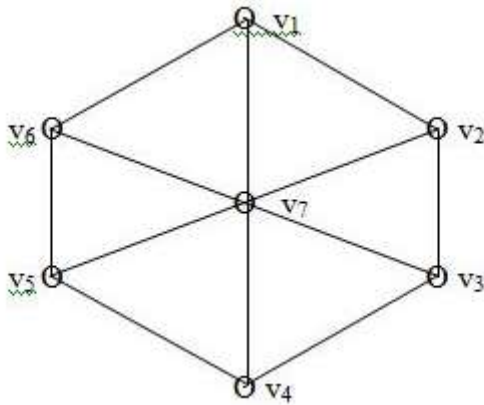


Fig – 2.1

Definition 2.2 Let G be the graph obtained by joining each vertex of a path P_n to a central vertex by edges. G is called fan graph and is denoted by F_n .

Theorem: 2.2 Fan graph F_n admits oblong difference mean prime labeling, if $n \not\equiv 0 \pmod{4}$ and $(n-1) \not\equiv 0 \pmod{4}$.

Proof: Let $G = F_n$ and let v_1, v_2, \dots, v_{n+1} are the vertices of G .

Here $|V(G)| = n+1$ and $|E(G)| = 2n-1$.

Define a function $f : V \rightarrow \{2, 6, 12, \dots, (n+2)(n+1)\}$ by

$$f(v_i) = i(i+1), \quad i = 1, 2, \dots, n+1.$$

For the vertex labeling f , the induced edge labeling f_{odmpl}^* is defined as follows

$$f_{odmpl}^*(v_i v_{i+1}) = (i+1), \quad i = 1, 2, \dots, n$$

$$f_{odmpl}^*(v_i v_{n+1}) = \frac{n^2+3n+2-i^2-i}{2}, \quad i = 1, 2, \dots, n-1$$

Clearly f_{odmpl}^* is an injection.

$$\begin{aligned}
 \text{gcin of } (v_1) &= \gcd \text{ of } \left\{ f_{odmpl}^*(v_1 v_2), \right. \\
 &\quad \left. f_{odmpl}^*(v_1 v_{n+1}) \right\} \\
 &= \gcd \text{ of } \left\{ 2, \frac{n^2+3n}{2} \right\} = 1. \\
 \text{gcin of } (v_{i+1}) &= \gcd \text{ of } \left\{ f_{odmpl}^*(v_i v_{i+1}), \right. \\
 &\quad \left. f_{odmpl}^*(v_{i+1} v_{i+2}) \right\} \\
 &= \gcd \text{ of } \{i+1, i+2\} \\
 &= 1, \quad i = 1, 2, \dots, \\
 &n-1 \\
 \text{gcin of } (v_{n+1}) &= \gcd \text{ of } \left\{ f_{odmpl}^*(v_n v_{n+1}), \right. \\
 &\quad \left. f_{odmpl}^*(v_{n+1} v_{n-1}) \right\} \\
 &= \gcd \text{ of } \{n+1, 2n+1\} \\
 &= \gcd \text{ of } \{(n+1), n\} = 1
 \end{aligned}$$

So, **gcin** of each vertex of degree greater than one is 1.

Hence F_n , admits oblong difference mean prime labeling.

Example 2.2 Oblong difference mean prime labeling of fan F_6 .

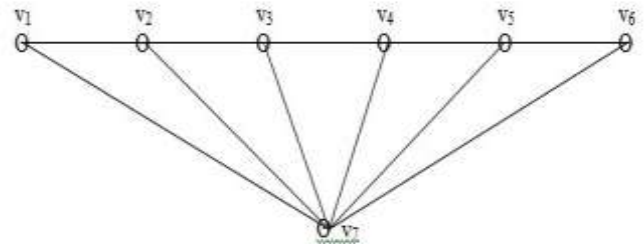


Fig – 2.2

Definition 2.3 Let G be the graph obtained by adding pendant edges to each vertex on the rim of cycle C_n of the wheel graph W_n . G is called helm graph and is denoted by H_n .

Theorem: 2.3 Helm graph H_n admits oblong difference mean prime labeling.

Proof: Let $G = H_n$ and let $v_1, v_2, \dots, v_{2n+1}$ are the vertices of G .

Here $|V(G)| = 2n+1$ and $|E(G)| = 3n$.

Define a function $f : V \rightarrow \{2, 6, 12, \dots, (2n+2)(2n+1)\}$ by

$$f(v_i) = i(i+1), \quad i = 1, 2, \dots, 2n+1.$$

For the vertex labeling f , the induced edge labeling f_{odmpl}^* is defined as follows

$$f_{odmpl}^*(v_i v_{i+1}) = (i+1), \quad i = 1, 2, \dots, n+1$$

$$f_{odmpl}^*(v_{i+2} v_{n+i+2}) = \frac{(n+i+2)(n+i+3) - (i+2)(i+3)}{2}, \quad i = 1, 2, \dots, n-2$$

$$f_{odmpl}^*(v_2 v_{n+1}) = \frac{n^2+3n-4}{2}$$

$$f_{odmpl}^*(v_{i+1} v_{2n+1}) = \frac{(2n+1)(2n+2)-(i+2)(i+1)}{2},$$

i = 1,2,-----

---,n

Clearly f_{odmpl}^* is an injection.

$$gcin \text{ of } (v_{i+1}) = \gcd \{ f_{odmpl}^*(v_i v_{i+1}), f_{odmpl}^*(v_{i+1} v_{i+2}) \}$$

$$= \gcd \{ i+1, i+2 \} = 1,$$

i = 1,2,-----

---,n

So, **gcin** of each vertex of degree greater than one is 1.

Hence H_n , admits oblong difference mean prime labeling.

Example 2.3 Oblong difference mean prime labeling of helm graph H_6 .

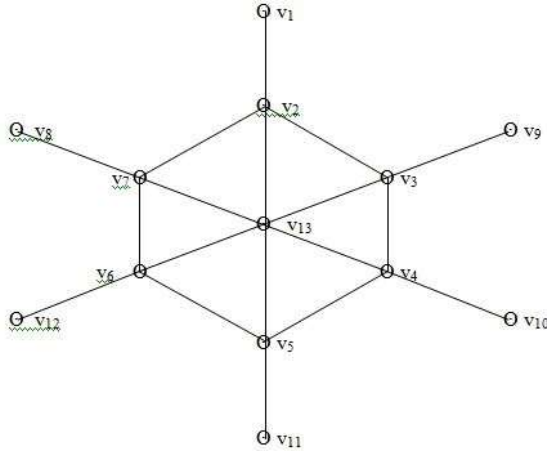


Fig – 2.3

Definition 2.4 Let G be the graph obtained by joining path P_n to a vertex of cycle C_n . G is denoted by the symbol $C_n(P_n)$ and is called tadpole graph.

Theorem: 2.4 Tadpole graph $C_n(P_n)$ ($n > 3$) admits oblong difference mean prime labeling, if $(n-1) \not\equiv 0 \pmod{4}$ and $(n-2) \not\equiv 0 \pmod{4}$.

Proof : Let $G = C_n(P_n)$ and let $v_1, v_2, \dots, v_{2n-1}$ are the vertices of G . Here $|V(G)| = 2n-1$ and $|E(G)| = 2n-1$. Define a function $f : V \rightarrow \{2, 6, 12, \dots, (2n)(2n-1)\}$ by $f(v_i) = i(i+1)$, $i = 1, 2, \dots, 2n-1$.

For the vertex labeling f , the induced edge labeling f_{odmpl}^* is defined as follows

$$f_{odmpl}^*(v_i v_{i+1}) = (i+1),$$

i = 1,2,-----,2n-2

$$f_{odmpl}^*(v_1 v_n) = \frac{n^2+n-2}{2}$$

Clearly f_{odmpl}^* is an injection.

$$gcin \text{ of } (v_1) = \gcd \{ f_{odmpl}^*(v_1 v_2), f_{odmpl}^*(v_1 v_n) \}$$

$$= \gcd \{ 2, \frac{n^2+n-2}{2} \} = 1.$$

$$gcin \text{ of } (v_{i+1}) = \gcd \{ f_{odmpl}^*(v_i v_{i+1}), f_{odmpl}^*(v_{i+1} v_{i+2}) \}$$

$$= \gcd \{ i+1, i+2 \} = 1,$$

i = 1,2,-----

---,n

So, **gcin** of each vertex of degree greater than one is 1.

Hence $C_n(P_n)$, admits oblong difference mean prime labeling.

Definition 2.5 Let G be the graph obtained by joining two copies of path P_n to two consecutive vertices of cycle C_n . G is denoted by the symbol $C_n(2P_n)$.

Theorem: 2.5 The graph $C_n(2P_n)$ admits oblong difference mean prime labeling.

Proof : Let $G = C_n(2P_n)$ and let $v_1, v_2, \dots, v_{3n-2}$ are the vertices of G . Here $|V(G)| = 3n-2$ and $|E(G)| = 3n-2$. Define a function $f : V \rightarrow \{2, 6, 12, \dots, (3n-2)(3n-1)\}$ by $f(v_i) = i(i+1)$, $i = 1, 2, \dots, 3n-2$.

For the vertex labeling f , the induced edge labeling f_{odmpl}^* is defined as follows

$$f_{odmpl}^*(v_i v_{i+1}) = (i+1), \quad i = 1, 2, \dots, 3n-3$$

$$f_{odmpl}^*(v_{2n-1} v_n) = \frac{3n^2-3n}{2}$$

Clearly f_{odmpl}^* is an injection.

$$gcin \text{ of } (v_{i+1}) = \gcd \{ f_{odmpl}^*(v_i v_{i+1}), f_{odmpl}^*(v_{i+1} v_{i+2}) \}$$

$$= \gcd \{ i+1, i+2 \} = 1,$$

i = 1,2,-----

---,3n-4

So, **gcin** of each vertex of degree greater than one is 1.

Hence $C_n(2P_n)$, admits oblong difference mean prime labeling.

Definition 2.6 Let G be the graph obtained by joining a path P_n to the apex vertex of fan F_n . G is called umbrella graph and is denoted by $U(n,n)$.

Theorem 2.6 Umbrella graph $U(n,n)$ ($n > 3$) admits oblong difference mean prime labeling, if $n \not\equiv 0 \pmod{4}$ and $(n-1) \not\equiv 0 \pmod{4}$.

Proof :Let $G = U(n,n)$ and let v_1, v_2, \dots, v_{2n} are the vertices of G .

Here $|V(G)| = 2n$ and $|E(G)| = 3n-2$.

Define a function $f : V \rightarrow \{2,6,12, \dots, (2n)(2n + 1)\}$ by

$$f(v_i) = i(i+1), \quad i = 1, 2, \dots, n+1.$$

For the vertex labeling f , the induced edge labeling f_{odmpl}^* is defined as follows

$$f_{odmpl}^*(v_i v_{i+1}) = (i+1), \quad i = 1, 2, \dots, 2n-1.$$

$$f_{odmpl}^*(v_i v_{n+1}) = \frac{n^2+3n+2-i^2-i}{2}, \quad i = 1, 2, \dots, n-1$$

Clearly f_{odmpl}^* is an injection.

$$gcin \text{ of } (v_1) = gcd \text{ of } \{ f_{odmpl}^*(v_1 v_2), f_{odmpl}^*(v_1 v_{n+1}) \}$$

$$= gcd \text{ of } \{ 2, \frac{n^2+3n}{2} \} = 1.$$

$$gcin \text{ of } (v_{i+1}) = gcd \text{ of } \{ f_{odmpl}^*(v_i v_{i+1}), f_{odmpl}^*(v_{i+1} v_{i+2}) \}$$

$$= gcd \text{ of } \{ i+1, i+2 \} = 1, \quad i = 1, 2, \dots, 2n-2.$$

So, $gcin$ of each vertex of degree greater than one is 1.

Hence $U(n,n)$, admits oblong difference mean prime labeling.

Theorem 2.7 Two Copies of cycle C_n ($n > 3$) sharing a common edge admits oblong difference mean prime labeling, if n is odd.

Proof :Let G be the graph and let $v_1, v_2, \dots, v_{2n-2}$ are the vertices of G .

Here $|V(G)| = 2n-2$ and $|E(G)| = 2n-1$.

Define a function $f : V \rightarrow \{2,6,12, \dots, (2n-1)(2n-2)\}$ by

$$f(v_i) = i(i+1), \quad i = 1, 2, \dots, 2n-2.$$

For the vertex labeling f , the induced edge labeling f_{odmpl}^* is defined as follows

$$f_{odmpl}^*(v_i v_{i+1}) = (i+1), \quad i = 1, 2, \dots, 2n-3.$$

$$f_{odmpl}^*(v_1 v_{2n-2}) = 2n^2-3n.$$

$$f_{odmpl}^*(\frac{v_{n+1}}{2} \frac{v_{3n-1}}{2}) = \frac{2n^2-n-1}{2}.$$

Clearly f_{odmpl}^* is an injection.

$$gcin \text{ of } (v_1) = gcd \text{ of } \{ f_{odmpl}^*(v_1 v_2), f_{odmpl}^*(v_1 v_{2n-2}) \}$$

$$= gcd \text{ of } \{ 2, 2n^2-3n \} = 1.$$

$$gcin \text{ of } (v_{i+1}) = gcd \text{ of } \{ f_{odmpl}^*(v_i v_{i+1}), f_{odmpl}^*(v_{i+1} v_{i+2}) \}$$

$$= gcd \text{ of } \{ i+1, i+2 \} = 1,$$

$$i = 1, 2, \dots, 2n-4$$

$$gcin \text{ of } (v_{2n-1}) = gcd \text{ of } \{ f_{odmpl}^*(v_{2n-2} v_1), f_{odmpl}^*(v_{2n-3} v_{2n-2}) \}$$

$$= gcd \text{ of } \{ 2n-2, 2n^2-3n \} = 1.$$

So, $gcin$ of each vertex of degree greater than one is 1.

Hence G , admits oblong difference mean prime labeling.

Definition 2.7 Let G be the graph obtained by joining two copies of cycle C_n by an edge. G is called dumbbell graph and is denoted by $D(n,n)$.

Theorem 2.8 Dumbbell graph $D(n,n)$ ($n > 3$) admits oblong difference mean prime labeling, if $n \equiv 0 \pmod{4}$ and $(n-3) \equiv 0 \pmod{4}$.

Proof :Let $G = D(n,n)$ and let v_1, v_2, \dots, v_{2n} are the vertices of G .

Here $|V(G)| = 2n$ and $|E(G)| = 2n+1$.

Define a function $f : V \rightarrow \{2,6,12, \dots, (2n)(2n + 1)\}$ by

$$f(v_i) = i(i+1), \quad i = 1, 2, \dots, 2n.$$

For the vertex labeling f , the induced edge labeling f_{odmpl}^* is defined as follows

$$f_{odmpl}^*(v_i v_{i+1}) = (i+1), \quad i = 1, 2, \dots, 2n-1.$$

$$f_{odmpl}^*(v_1 v_n) = \frac{n^2+n-2}{2}.$$

$$f_{odmpl}^*(v_{n+1} v_{2n}) = \frac{3n^2-n-2}{2}.$$

Clearly f_{odmpl}^* is an injection.

$$gcin \text{ of } (v_1) = gcd \text{ of } \{ f_{odmpl}^*(v_1 v_2), f_{odmpl}^*(v_1 v_n) \}$$

$$= gcd \text{ of } \{ 2, \frac{n^2+n-2}{2} \} = 1.$$

$$gcin \text{ of } (v_{i+1}) = gcd \text{ of } \{ f_{odmpl}^*(v_i v_{i+1}), f_{odmpl}^*(v_{i+1} v_{i+2}) \}$$

$$= gcd \text{ of } \{ i+1, i+2 \} = 1, \quad i = 1, 2, \dots, 2n-2.$$

$$gcin \text{ of } (v_{2n}) = gcd \text{ of } \{ f_{odmpl}^*(v_{2n-1} v_{2n}), f_{odmpl}^*(v_{n+1} v_{2n}) \}$$

$$= gcd \text{ of } \{ 2n, \frac{3n^2-n-2}{2} \} = 1.$$

So, $gcin$ of each vertex of degree greater than one is 1.

Hence $D(n,n)$, admits oblong difference mean prime labeling.

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