Software Modelling of Coupler Curve Generation using Planar Five Bar Constrained Mechanism

Authors
Abhay K. Abhyankar¹, S.Y.Gajjal²
¹,²NBN Sinhgad School of Engineering, Ambegaon, Pune, Maharashtra, India
Email: abhyankar.abhay@gmail.com

ABSTRACT
Five bar planar mechanism has degree of freedom two. Hence constraint should be provided to reduce the degree of freedom to one. Thus it becomes a tractable problem. The software model of this mechanism helps the user to visualize the motion curves generated. In the simulation of planar five bar mechanism, the objective is to calculate the positions of all the links which are rigid bodies. There can be two input links. By adding a new constraint to conventional five bar motion generation model, the degree of freedom is reduced to one. This five bar constrained mechanism generates a family of curves. The software model allows the user to actually input the mechanism and generate the motion curve. The model indicates the locking positions and user can modify the dimensions.

Keywords: Five Bar Planar mechanism, Software Model, Coupler curve, Constraints, Motion Generation

1. INTRODUCTION

1.1 Motion Curve Generation
The coupler curve generation is related to the dimensions of the mechanism and the constraints. In the synthesis of mechanism, we are given some positions which are to be achieved by the mechanism to be designed. In some situations we are given three or four positions of a link which must be achieved. In this software modeling, the entire motion curve will be given and the mechanism should generate the same motion curve for some point on its linkage. The software model is designed in such a way that the user can chose the mechanism dimensions and orientation which will generate a similar curve from the library.

The software model also allows the user to design a five bar mechanism of any dimensions and orientations, and the software will test it during simulation. The generated curve is simulated if the mechanism does not come across locking situation. The user will be allowed to change dimensions, positions as well as orientation. The velocity of the desired point is also calculated and graphed.

When the motion curve is given, user is required to get the five bar mechanism from the library to generate the similar curve as close as possible. The software model will calculate the dimensions for the five bar mechanism, which will generate the given curve within the tolerance limit.
1.2 Literature Survey
As a survey, different papers were referred and studied. In this area of five bar planar mechanisms, Balli and Chand [1] introduced many methods for motion generation. In addition they have also elaborated method for synthesizing five bar mechanisms by using transmission angle control. Wang and Yen [3] have discussed the precise approach for synthesizing with five prescribed precision positions. Basu and Farhang [4] introduced a mathematical formulation for the approximate analysis and design of two input, small crank five bar mechanisms for function generation. Lin and Chang[5] modified pole method for synthesis planar, geared five bar function generators. Different softwares for modelling were also checked to solve the similar problems. Ge and Chen[6] introduced software based method for path synthesis. The authors have also given the effect of the ling length, crack angles and gear tooth ratio on the motion of geared five bar linkage. Cheng and Trang[7] explained the web based mechanism and analysis of five geared planar synthesis. Some authors have designed mechanical automata. In the first stage the motion approximation on the input sequence is done in relation with the geometric and motion constraints of mechanical parts. In stage two they generate the layout by solving sizing parameters and spatial layout of all the elements. So by referring all these, we are developing a Simulation Model for Coupler Curve Generation using Five Bar Planar Mechanism with Rotation Constraint. In which, the model will allow the user to actually input the mechanism and generate the motion curve. During the survey it is noticed that no software model is available for simulating the working of this mechanism.

1.3 Related Work
For analysis and design of mechanisms, computer software can be used. This fact was recognized in the earlier days of computers.[Freudenstein 1954].The approaches used by the software systems can be different, it can be function oriented implementation or shape oriented implementation. The relation among the positions will be represented as nodes on the graph. In several design methods, a specified input motion is transformed into output motion. Chiou and Sridhar [1999] used a library of mechanisms. These are used as basic building blocks along with symbolic matrices which represented the library. The system decomposes an intended function into simple functions so that the match will be found in the library. A mechanism can be selected from the parameterized set as per the prior knowledge of the motion. Zhu et al.[2012]. This approach can handle linear, ellipsoidal or circular motions. In this work, we are using a sampling based approach which generates a representative set of motions using a five bar mechanism.

1.4 Motivation and Scope of work
In conventional motion generation method, user can calculate the mechanism dimensions to achieve the required set of the link positions. Two or three link orientations will be given and we can design the remaining links of the four bar mechanism. These methods will not assure about the entire motion
curve. In this work, the entire motion curve of a point is given to us. Hence the conventional method can not be used. The five bar mechanism has degree of freedom as two. It allows us to adjust the speed and directions of the two input links. This will generate a wide variety of curves, which do not have any mathematical equation. We restrict our scope for the input links rotating in opposite directions with same angular speeds. Here a constraint is incorporated in the five bar mechanism to reduce the degree of freedom to one. There will be two input links. The constraint is provided in such a way that both the input links rotate with same angular velocity, either in same direction or opposite direction.

The dependency of the motion of point on the extension of a coupler link on the dimensional and orientation parameters can result in variety in the shapes of the coupler curves. Unavailability of dedicated software for this purpose worked as the basic motivation for this software model.

2.MATHEMATICAL MODEL FIVE BAR PLANAR MECHANISM

2.1 Complex Number model

Complex number model allows us to establish the relation between the length and the orientation of each link. This model will help us to calculate the position of the required point. Fig. 1 shows the five bar mechanism along with the lengths and current position angles.

![Five bar planar mechanism with dimensions](image)

**Terminology:**

OP is fixed link.

OA and PB are input links.

01 and 02 are initial orientation angles of links OA and PB respectively.

ϕ1 and ϕ2 are angles made by links AC and BC

r1 and r2 are lengths of input links OA and PB.

L1 and L2 are lengths of auxiliary links AC and BC.

ω1 and ω2 are angular velocities of OA and PB such that ω1 = -ω2

\[
OA = a = r_1 \ e^{i\phi_1} \quad \text{..... (i)}
\]

\[
OP = p \quad \text{..... (ii)}
\]

\[
b = p + r_2 \ e^{i\phi_2} \quad \text{..... (iii)}
\]

\[
c = a + AC = a + L_1 \ e^{i\phi_1} \quad \text{.... (iv)}
\]

Also \(c = b + BC = b + L_2 \ e^{i\phi_2} \quad \text{.... (v)}\)

Equating c from (iv) and (v)

\[
a + L_1 \ e^{i\phi_1} = b + L_2 \ e^{i\phi_2}
\]

\[
r_1 e^{i\phi_1} + L_1 e^{i\phi_1} = p + r_2 e^{i\phi_2} + L_2 e^{i\phi_2}
\]

Equating real and imaginary parts

\[
r_1 \cos(\phi_1) + L_1 \cos(\phi_1) = p + r_2 \cos(\phi_2) + L_2 \cos(\phi_2)
\]

\[
r_1 \sin(\phi_1) + L_1 \sin(\phi_1) = r_2 \sin(\phi_2) + L_2 \sin(\phi_2)
\]
For the given mechanism \( r_1, r_2, L_1, L_2, p, \theta_1, \theta_2 \) are all known. The only unknowns are \( \phi_1 \) and \( \phi_2 \).

Hence we get simultaneous equations as
\[
\begin{align*}
L_1 \cos(\phi_1) - L_2 \cos(\phi_2) &= \alpha \\
L_1 \sin(\phi_1) - L_2 \sin(\phi_2) &= \beta
\end{align*}
\]
Solving we get
\[
\cos(\phi_2) = \frac{-Q + \sqrt{Q^2 - 4PR}}{2P}
\]
where
\[
\begin{align*}
\alpha &= -r_1 \cos(\theta_1) + p + r_2 \cos(\theta_2) \\
\beta &= -r_1 \sin(\theta_1) + r_2 \sin(\theta_2) \\
\chi &= \frac{(L_1^2 - L_2^2 - \alpha^2 - \beta^2)}{2L_2} \\
P &= \beta^2 - \alpha^2 \\
Q &= 2\alpha^2 \chi \\
R &= \beta^2 - \chi^2
\end{align*}
\]
For the existence of solution
\[
Q^2 - 4PR > 0
\]
And numerically
\[
| -Q + \sqrt{Q^2 - 4PR} | < |2P|
\]
Which results in basic condition that
\[
L_2 > L_1 \text{ and some more.}
\]

2.2 Constant length geometrical approach

While designing the mathematical model, we use the constant length approach to find the coordinates of the points. It uses the mathematical formulae for the length and circles. The intersection of these circles with each other and the lines will help us to determine the coordinates of the points.

\[
\begin{align*}
x_2 - x_1 &= r_1 \cos(\theta_1) \\
y_2 - y_1 &= r_1 \sin(\theta_1) \\
x_4 - x_3 &= r_2 \cos(\theta_2) \\
y_4 - y_3 &= r_2 \sin(\theta_2)
\end{align*}
\]
Circles with centers 2 and 4 having radii \( L_1 \) and \( L_2 \), will intersect at point 5.
\[
\begin{align*}
(x - x_2)^2 + (y - y_2)^2 &= L_1^2 \\
(x - x_4)^2 + (y - y_4)^2 &= L_2^2
\end{align*}
\]
Solving these two equations for \( x \) and \( y \)
We come across following steps
\[
2(x - x_2) k_1 = k_2 + 2y k_3
\]
Where \( k_1 = x_2 - x_4 \)
\[
k_2 = (L_2 - L_1)^2 - k_1^2 + y^2 - y_4^2 \\
k_3 = y_2 - y_4
\]
finally the quadratic equation
\[
a_1 y^2 + b_1 y + c_1 = 0
\]
where \( a_1 = (k_3/k_1)^2 + 1 \)
\[
b_1 = (k_2/k_1)(k_3/k_1) - y_2 \\
c_1 = (k_2/2k_1)^2 - L_1^2 + y_2^2
\]
The further conditions will be
\[
b_1^2 - 4a_1 c_1 > 0 \quad \ldots \quad \text{necessary to get real value of } y.
\]
This condition is similar to the condition we got in the previous approach.

3. SOFTWARE MODEL

The software model for the planar five bar mechanism is developed using C language with graphic programming. A very simple and user friendly interface is provided. User will work with his/her own idea and can input the mechanism. There will be one line help available for the user. Both keyboard and mouse have their own importance while handling the software model.

It provides the user following facilities

**Input:** Five bar mechanism with lengths and orientation. This will be done only by using the mouse as the points are to be clicked.

**Edit:** User can modify the parameters. Each point of the mechanism can be modified, its position can be changed. Thus only two lengths will changed at a time.

**Save:** The configuration can be saved. There is an automatic file naming provided.

**Simulate:** The mechanism can rotated by the use of simple key stroke, one position at the time, leaving the trace of the earlier point. Thus the user can see the coupler curve for points C and D

**Database:** As the mechanism with all the points, lengths will be stored in the file, user can select any configuration from the database

**Input:** Motion Curve can given as a set of consecutive points.

**Matching:** Selecting the mechanism to suit the motion curve and edit to get the mechanism to generate the same curve within tolerance limit. This will be done by the user in the manual manner.

Fig. 3 General user interface for the simulation software

Refer to Fig. 3, which shows the general layout of a simple system designed for simulation of five bar mechanism.

The user will be guided to draw the five bar mechanism, starting from the two input links 1-2 and 3-4. Then the user will have to give the point to be traced, point 6. For this discussion, we connect points 2 and 6, and get point 5, which is midpoint of 2-6. Thus we get the auxiliary links 2-5 and 4-5. As the user has already given the orientations of the links 1-2 and 3-4, the simulation tool now can simulate the movement of the mechanism when 1-2 and 3-4 rotate with same angular velocities but in opposite direction. For the user viewing advantage the increment in the angle is 5 degrees so that user can visualize the displacement difference in the successive points on the curve and can get the idea about the velocity as the time interval is constant. More the distance between the successive points, more will be the linear velocity.
4. RESULTS AND DISCUSSION

The software model is developed for the five bar planar mechanism with constraints. Both the input links are given the same angular velocities. The sense of rotation of input links is opposite to each other. The simulation tool (software) revealed some interesting results in the form of the curves. Effect of the change in the lengths of the links and initial orientation is directly visible on the shape of the curves. The user will have the facility to modify each of the parameter. The change has also affected the velocity at different points on the curve. The motion generation can be studied very effectively using this tool.

The standard four bar mechanisms have a limitation on the generation of the motion curves. All the results which are obtained can be saved for the further use. The user has full freedom to vary any parameter and see the effect. When the entire motion is not possible, the tool generates the error message and the limiting position. The mechanism in such case will keep on oscillating between the limiting positions. The tool has limitations due to the basic constrains which are applied to the rotation of input links. The study will continue further when the input links are provided with different angular velocities. It will also generate more curves of different shapes. The input link velocities can be easily managed by providing the gears with tooth ration 1:1. This will provide same angular velocity and opposite direction.

Five bar planar mechanism with constraints have been explored in recent future but motion generation and coupler curves need to be explored.
to a large extent. The results shown in this software tool show that the curves can contain straight portions, same path tracing, repeating the same point as well as multiple self intersections.

In the next version of the software model, the rotational constraint will be modified to different ratios of angular velocities of input links. In such a case user is free to decide the ratio of the angular velocities and their direction.

4.1 Step by step variation of point 1 : Results

**Fig. 6.** Original Configuration

Step by step left movement of fixed end 1 is shown next. The original mechanism is retrieved from the database and user is allowed to move only point 1 in any of left, right, up, down direction. This will facilitate the observation related to shape of curve.

**Fig. 7** Left movement of fixed end 1: step1

**Fig. 8** Left movement of fixed end 1: step2

**Fig. 9** Left movement of fixed end 1: step3

4.2 Curves generated: Results

The different curves generated by the simulator are shown below.

**Fig. 10** Some of the curves generated using the simulation software
4.3 Extreme variation of point 1: Results

Fig. 10 (a) Original Curve

Fig. 10 (b) Leftmost position of point 1

Fig. 10 (c) Rightmost position of point 1

Fig. 10 (d) Topmost position of point 1

Fig. 10 (e) Lowermost position of point 1

Fig. 10: Extreme variation in shape of curve

(* Refer to annexure for the other movements of point 1 in some other configuration of the five bar mechanism. It will show the step by step changes in the shape of the curve)

5. CONCLUSION

Observing the results obtained from the software model we find that the five bar planar mechanism with rotational constraint is capable of generating a wide variety of curves. The different curves are generated by varying the parameters associated with the mechanism namely, the lengths of the links and initial orientation of the driving links. At the same time we remember that the rotational constraint on
the five bar mechanism of same angular velocity and opposite direction can be changed to different ratios of angular velocities and even in same direction. These changes will be implemented in the next version of this software model. These changes will result in even larger number of curves which can find their applications in the areas like material handling, character animation using motion captured curves.

If the concept is used effectively, it can be used in industries like embroidery, pattern painting etc.

REFERENCES


ANNEXURE:

* Different curves obtained in steps by movement of point 1

- Right movement of fixed end 1: step1
- Right movement of fixed end 1: step2
- Right movement of fixed end 1: step3
- Up movement of fixed end 1: step1
- Up movement of fixed end 1: step2
- Up movement of fixed end 1: step3
- Up movement of fixed end 1: step4
- Up movement of fixed end 1: step5
- Down movement of fixed end 1: step1
- Down movement of fixed end 1: step2
- Down movement of fixed end 1: step3
- Down movement of fixed end 1: step4