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Stability Analysis of Nonlinear Systems with Transportation Lag

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ABSTRACT
This paper formulates an algebraic criterion to analyze stability of nonlinear systems with transportation lag that arise in engineering systems. Here the stability can be analyzed with the help of complex polynomials. In the proposed scheme which is termed as Sign Pair Criterion, Routh-like table is developed and the elements in the first column are utilized to get the stability results. Here the real and imaginary parts of the given complex polynomial are separated to form the first two rows of Routh –like table. The Routh multiplication rule is applied to find all the remaining elements of the table.
Keywords- *Complex polynomials, Stability analysis, Nonlinear Systems, Transportation Lag , Sign pair criterion*

1. INTRODUCTION

Most of the components and actuators found in physical systems have nonlinear characteristics. Some of the devices have moderate nonlinear characteristics, or the nonlinear properties occur only if they are driven into certain operating region. For these types of devices linear system models may give quite accurate analytical results over a relatively wide range of operating conditions.

In control systems, nonlinearities can be classified as incidental and intentional. Incidental nonlinearities are those which are inherently present in the system. Common examples are saturation, dead-zone, coulomb friction, stiction, backlash etc. The intentional nonlinearities are those which are deliberately inserted in the system to modify system characteristics. The most common example of this type of nonlinearity is a relay.

Nonlinearities are common in control systems, particularly where high gains are required and saturated amplifiers or on-off actuators are used. The sources of transportation lag are not always apparent but their presence is obvious. Both nonlinearities and transportation lag seriously deteriorate system stability and also the stability analysis becomes much more difficult. First the mathematical description of the system is linearised by using a describing function to represent the nonlinearity.

Many of the engineering systems like thermal process, space control systems, machine tool chattering as well as physical and biological systems involve transportation lag and hence governed by differential equations [1], [2].

The stability problem of time–delays in various engineering system has been analyzed by Olgac & Sipahi [3]. A new method utilizing the method of steps and numerical inversion of Laplace transforms for the stability analysis of delay differential systems is proposed by Kalmar-Nagy [4]. Stability analysis of uncertain systems with multiple time-varying delays is done by Sun et al [5]. The non rational time delay function $e^{-s\tau}$ can be represented as a rational function using Pade approximation as explained in Richard & Robert [1] for analysis and design purposes. The stability equation method introduced by Khan and Thaler as mentioned in [6] can be used for stability analysis of both linear and nonlinear systems.

In the proposed scheme, the 'Modified Routh's table' is formed after separating the real and imaginary parts of the characteristic equation by substituting s= 'jω'. Applying Routh–Hurwitz criterion, the number of the roots of the characteristic equation having positive real part can be revealed. The proof for the Sign Pair Criterion is given in [7].

2. PROPOSED SCHEME- SIGN PAIR CRITERION

In this paper, an algebraic scheme is proposed for the analysis of stability of nonlinear systems with transportation lag. With the substitution of s = jω , the real and imaginary parts of the characteristic equations , are extracted separately and their coefficients are entered suitably in the first-two rows of Routh-like table to observe the system stability. The formulated stability criterion is termed as 'Sign Pair Criterion' (SPC). In this procedure, it can be noted that the Routh-like table contains only real elements.

Let the characteristic equation of the system be

$$C(s) = s^n + (a_1 + j\, b_1)s^{n-1} + (a_2 + j\, b_2)s^{n-2} + \cdots + (a_k + j\, b_k) = 0$$

Substituting of s = jω,

$$C(j\omega) = (j\omega)^n + (a_1 + j\, b_1)(j\omega)^{n-1} + (a_2 + j\, b_2)(j\omega)^{n-2} + \ldots + (a_k + jb_k) = 0$$

$$= R(\omega) + jI(\omega) = 0$$

Where

$$R(\omega) = (A_0\omega^n + A_1\,\omega^{n-1} + A_2\,\omega^{n-2} + \cdots + A_n)$$

$$I(\omega) = (B_0\omega^n + B_1\,\omega^{n-1} + B_2\,\omega^{n-2} + \cdots + B_n)$$

Using the coefficients of above polynomials, the second form of Routh-like table can be formulated as

A_0	A_1	A_2	...	A_n
B_0	B_1	B_2	...	B_n
c_0	c_1	c_2	...	
d_0	d_1	d_2	...	
e_0	e_1	e_2	...	
f_0	f_1	f_2	...	
g_0	g_1	...	...	
.	.	.		
.	.	.		
.	.	.		

2.1Algorithm for Sign Pair Criterion

- If the first element in the first row is negative, multiply the full row elements by -1.
- If the first element in the second row is zero, interchange first and second rows and multiply all elements in the second row by −1.1
- Follow the Common Routh's multiplication rule to get the complete table with '2n+1' rows.
- If any element of the first column starting from third, comes zero, it is replaced by a small value +0.01.
- If all the elements in a row become zero, then the auxiliary polynomial is formed using the previous row elements and differentiated once; the coefficients of this modified polynomial are entered instead of zeros and the table is completed by applying the Routh multiplication rule.
- Get 'n' sign pairs using the first column elements starting from second row.

The sign pairs are developed as

$P_1 = (B_0, c_0)$

$P_2 = (d_0, e_0)$,$P_3 = (f_0, g_0)$ … .P_n. .

According to the this scheme SPC , it is ascertained that each element of all the pairs has to maintain the same sign for the roots of characteristic equation to lie on the left hand side of s-plane for stability. The proof of the criterion is given in [7].

3. MODEL OF TRANSPORTATION LAG

Transportation lag is very often encountered in various technical systems, such as electric, pneumatic and hydraulic networks, chemical processes, long transmission lines, robotics, thermal process, distillation process, space control systems, machine tool chattering etc. The existence of pure time lag, regardless if it is present in the control or/and state, may cause undesirable system transient response or even instability.

A time delay in a feedback system introduces an additional phase lag and results in a less stable system. Therefore as pure time delays are unavoidable in many systems. It is often necessary to reduce the loop gain in order to obtain a stable response. But the cost of stability is the resulting increase in the steady state error of the system as the loop gain is reduced. A simple pure time-delay system is shown in Figure 1.

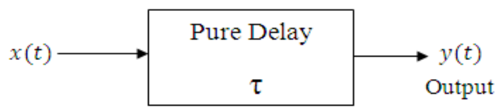

Figure 1 Time-delay element with τ (seconds)

Due to this delay, the input will be delayed and the output is

$$y(t) = x(t - \tau), \qquad t > \tau$$

Applying Laplace transformation to the above equation $y(s) = e^{-s\tau}\, x(s)$

Thus, the transfer function for the system shown in Figure 1 is

$$\frac{y(s)}{x(s)} = e^{-s\tau}$$

Where

$$e^{-sT} = e^{-j\omega T} = \cos \omega T + j \sin \omega T$$

This results an equation with complex coefficients. The nonlinearity of the system is linearised by describing function.

4. EFFECT OF TRANSPORTATION LAG

When the system contains a transportation lag, the characteristic equation is modified.

4.1 Systems with an Ideal Relay

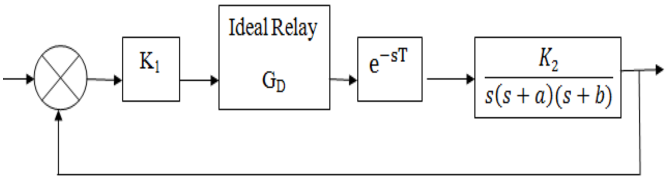

Figure 2. A system with transportation lag and an ideal relay

Where G_D is the describing function of the nonlinearity and $G_D = 4\,H/\,\pi\,A$; 'H' is the output level of the relay and 'A' is the signal amplitude into the relay[9],[10]. The characteristic equation for the above system as given in [8] is given below.

$$F(s) = s^3 + (a+b)\,s^2 + abs + K_1K_2G_D\mathrm{e}^{-sT} = 0$$

By substituting '$s = j\,\omega$'

$$e^{-sT} = e^{-j\omega T} = cos\ \omega T - j\ sin\ \omega T$$

$$F(j\omega) = R(\omega) + j\ I(\omega) = 0$$

$$F(j\omega) = -(a+b)\omega^2 + K_1K_2G_D cos\omega T + j(-\omega^3 + ab\omega - K_1K_2G_D sin\omega T)$$

$$F(j\omega) = -C_1\,\omega^2 + D_1 + j\,(-\omega^3 + C_2\,\omega - D_2) = 0$$

Where

$C_1 = a + b$

$C_2 = ab$

$D_1 = K_1K_2\,G_D \cos\omega\mathrm{T}$

$D_2 = K_1K_2\,G_D \sin\omega\mathrm{T}$

The Routh-like table is formed as

0	$-C1$	0	$D1$
-1	0	$C2$	$-D2$
$-C1$	0	$-D1$	
$+\alpha$	$(C1C2 - D1)/C1 - D2$		
$+C1C2 - D1$	$-C1D2$		
$+(C1C2 - D1)/C1$	$-D2$		
$+\alpha$			

The stability conditions for the above system as per sign pair criterion are

$C_1 > 0$ And

$C_1C_2 - D_1 > 0, or\ \ C_1C_2 > D_1$

4.2 Systems with Relay, Deadzone & Hysteresis

If the nonlinearity includes a hysteretic effect, its describing function is complex, i.e. $N_1 + jN_2$ [9],[10]. When the negative feedback system uses a relay with dead zone and hysteresis combined with transportation lag as mentioned by [7], the block diagram representation of the system is given below.

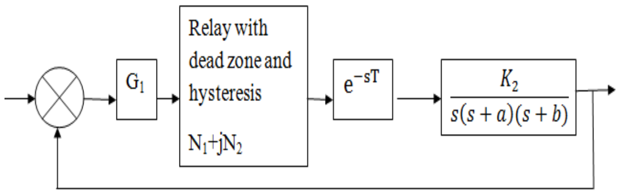

Figure 3. A System with relay, deadzone and hysteresis

The characteristic equation for $G_1 = K_1$ is given by

$$F(s) = s^3 + (a+b)s^2 + abs + K_1K_2(N_1 + jN_2)e^{-sT} = 0$$

By substituting '$s = j\,\omega$'

$e^{-sT} = e(-j\omega T) = \cos \omega T - j \sin \omega T$ and

$F(j\omega) = R(\omega) + j\, I(\omega) = 0$

$F(j\omega) = -(a+b)\,\omega^2 + K_1K_2(N_1 \cos \omega T +$
$N2 \sin\omega T\,)+$

$j\,(-\omega^3 + ab\omega + K_1K_2(N_2 \cos \omega T -$
$N_1 \sin \omega T) = 0$

$F(j\omega) = -C_1\,\omega^2 + D_1 + j\,(-\omega^3 + C_2\omega - D_2\,) = 0$

where

$C_1 = a + b$

$C_2 = ab$

$D_1 = K_1\,K_2\,(N_1 \cos \omega T + N_2 \sin \omega T)$

$D_2 = -K_1K_2(N_2 \cos \omega T - N_1 \sin \omega T\,)$

The Routh-like table is for the equation is found to be same as the previous one and the stability condition for the above system can be written as

i. $C_1 > 0$ and
ii. $C_1C_2 - D_1 > 0, or\ \ C_1C_2 > D_1$

4.3 Aircraft Pitch Control

Consider the pitch control of an aircraft as shown in the figure [7]. Here the nonlinearity and transportation lag are in the actuator and associated mechanisms. 'S_i' and 'S_r' are the sensitivities of the integrating gyro and rate gyro respectively.

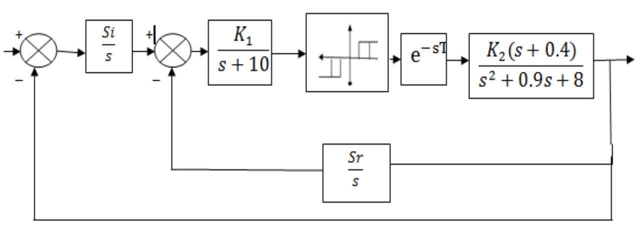

Figure 4. An aircraft pitch control system

$$F(s) = s^4 + 10.9s^3 + 17\,s^2 + 80s + K_1K_2(N_1 + jN_2)(s+0.4)(S_i + S_r)e^{-sT} = 0$$

This equation is also of the form of complex coefficients and the proposed sign pair criterion can be applied for stability analysis if the values of constants are known for the given system.

5. CONCLUSIONS

In this paper, the stability investigation of non linear systems with transportation lag has been performed with the help of the proposed Sign pair criterion (SPC). MATLAB program for the scheme is developed which makes the computation very simple and effective compared to other existing methods for stability analysis systems with delay and nonlinearity

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REFERENCES

1. C.D. Richard and H. B. Robert, *Modern Control Systems*, Dorling Kindersley (India) Pvt. Ltd., Pearson Education, 2008.
2. J. Nagrath and M. Gopal, *Control Systems Engineering*, New Age International (P) Ltd, New Delhi, India, 2007
3. N. Olgac and R. Sipahi, "An Extant Method for the Stability Analysis of Time – Delayed Linear Time – Invariant Systems," *IEEE Trans. Automatic Control.*, vol.47, No. 5, pp. 793 – 797, 2002.
4. T. Kalmar-Nagy, "Stability analysis of delay-differential equations by the method of steps and inverse Laplace transform", *Differential Equations and Dynamical Systems*, vol.17, no.1, pp.185-200,2009.

5. Y. J. Sun, J. G. Hsieh and H. C. Yang, “On the stability of uncertain systems with multiple time – varying delays”, *IEEE Transactions on Automatic Control*, vol.42, no.1, pp.101- 105, 1997.
6. Y.T. Tsay, B. C. Wang & K.W. Han, “Stability analysis of nonlinear control systems with characteristic equations having complex coefficients”, *Journal of the Franklin Institute*, vol. 97, no.3, pp.179-186, 1974.
7. K. Sreekala and S. N. Sivanandam, “Modified Routh’s Table for the Stability Analysis of Linear Systems Having Complex Coefficient Polynomials”, *International Review of Mechanical Engineering*, Vol.6,No.6, pp.1213-1216,2012.
8. Y. Y. K. Chen, K. W. Han and J.T. George, “Analysis of nonlinear control systems with transportation lag” , *IEEE Transactions on Industry and General Applications*, vol.7, no.5, pp. 576-579,1971.
9. R. Sridhar, “A general method for deriving the describing function for a certain class of nonlinearities”, *IRE Transactions on Automatic Control*, *vol.5*, no.2, pp.135-141,1960.
10. K. Ogatta, *Modern Control Engineering*, Prentice Hall, USA. 2001