IJETST- Vol.||02||Issue||07||Pages 2909-2914||July||ISSN 2348-9480 2015

Open access Journal International Journal of Emerging Trends in Science and Technology

On b-δ-continuous functions

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Abstract: *In this paper, we introduce and study two new of functions called b-δ- continuous function by using the notions of b-δ-open sets and b-δ-closed sets.*
Keywords: *δ - open sets, b-open set, b-δ-open sets, b-δ-closed sets, b-δ-continuous function.*

1. Introduction

Generalized open sets play a very important role in general topology and they are now the research topics of many topologists worldwide. Indeed a significant theme in general topology and real analysis concerns the various modified forms of continuity, separation axioms etc., by utilizing generalized open sets. In 1961, Levine [12] introduced the concept of weak continuity as a generalization of continuity Latter in 1963, Levine [13] also introduced the concept of semi open sets in topological space. Since then numerous applications have been found in studying different types of continuous like maps and separation of axioms.. In 1966, Hussain [11] introduced almost continuity as another generalization of continuity and Andrew and Whittley [2] introduced the concept of closure continuity which is stronger than weak continuity. The concept of δ -interior, δ -closure, θ-interior and θ-closure operators were first introduced by Velico [26] in 1968, for the purpose of studying the important class of H-closed spaces. These operators have since been studied intensively by many authors.

In 1970, Levine [14] initiated the study of generalized closed sets, i.e., the sets whose closure belongs to every open superset and defined the notation of $T_{1/2}$ space to be one in which the closed sets and generalized closed sets coincide. In 1980, the notion of δ -continuous function was introduced and studied by Noiri [20]. Latter in 1982, Mashhour.et.al, [15] introduced the concept of pre open sets. In 1986, the notion of semi-pre open set was introduced by Andrijevic[3]. Latter in 1996, Andrijevic [4] introduced a class of generalized open sets in a topological space, so called b-open sets. The class of b-open sets is contained in the class of semi preopen sets and contains all semi-open sets and pre-open sets. In 2003, Ganguly.et al[10] introduced the notion of strongly δ -continuous function in topological spaces. Latter, El.Atik [9] introduced and studied the notion of b-continuous function. He also introduced and studied a new class of functions called b-irresolute function. This notion has been studied extensively in recent years by many topologists. The notions of b-δ -closed Sets was introduced and studied by Padmanaban [22]. The purpose of this paper is to introduce a new class of functions called b-δ- continuous functions. We investigate some of the fundamental properties of this class of function. We recall some basic definitions and known results.

2. Preliminary

Throughout this paper, spaces (X, τ) and (Y, σ) (or simply X and Y) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a space (X, τ). We denote closure and interior of A by cl(A) and int(A), respectively. The set A is said to be regular open (resp. regular closed) [25] if A = int(cl(A))(resp. A = cl(int(A))).The family of all regular open(resp. Regular closed) sets of (X, τ) is written by RO(X, τ) (resp. RC(X, τ)).This family is closed under the finite intersections (resp. finite unions). The δ –closure of A [26] is the set of all x in x such that the interior of every closed neighbourhood of x intersects A non trivally. The δ –closure of A is denoted by cl_δ (A) or δ-cl(A) . The δ –interior of a subset A of X is the union of all regular open sets of X contained in A and is denoted by δ-int(A).The subset A is called δ-open if A= δ-int (A). i.e, a set is δ-open if it is the union of regular open sets.The complement of δ-open set is δ-closed. Alternatively, a set A ⊂ X is called δ-closed if A= δ-cl(A), where δ-cl(A) = {x ∈ X : int(cl(U)) ∩ A ≠ φ, U ∈ τ and x ∈ X}.

A subset A is said to be b-open [4] if A ⊂ cl(int(A)) ∪ int(cl(A)).The complement of b-open is said to be b-closed. The intersection of b-closed sets of X containing is called b-closure of A and denoted by b-cl(A).The union of all b-open sets of X contained in A is called b-interior and is denoted by b-int(A).The subset A is b-regular if it is b-open and b-closed. The family of b-open (b-closed, b-regular) sets of X is denoted by BO(X) (resp. BC(X), BR(X)) and family of all b-open (b-regular) sets of X containing a point $x \in X$ is denoted by BO(X, x) (resp.BR(X, x)).

Let A be a subset of a topological space (X,τ) . A point x of X is called a b-δ-cluster point [22] of A if int (b-cl(U)) ∩ A ≠ φ for every b-open set U of X containing x. The set of all b-δ-cluster point of A is called b-δ-closure of A and is denoted by b-δ-cl(A). A subset A of a topological space (X, τ) is said to be b-δ-closed, if A = b-δ-cl(A). The complement of a b-δ-closed set is said to be b-δ-open set.The b-δ-interior of a subset A of X is defined as the union of all b-δ-open sets contained in A and is denoted by b-δ-int(A). Alternatively, a point x in X is called b-δ-interior point of A, if there exists a b-open sets containing x such that int(b-cl(U)) ⊆ A. The set of all b-δ-interior points of A is called b-δ-interior of A. The family of all b-δ-open sets of the space (X,τ) is denoted by BδO(X) and the family of all b-δ-closed sets of the space (X, τ) is denoted by BδC(X).

A function f: (X, τ) → (Y, σ) is said to be

1. Open [7] if f (U) is open in (Y, σ) for every open set U of (X, τ),
2. α -open [16] if f(U) is α-open in (Y, σ) for every open set U of (X, τ),
3. β -open[1] if f(U) is image of each open set U of (X, τ) is a β -open set,
4. b-open [9] if f(U) is b-open in (Y, σ) for every open set U of (X, τ),
5. b-continuous [9]] if for each x ∈ X and each open set V of Y containing f(x), there exists U ∈ BO(X,x) such that f(U) ⊆ V,
6. δ -continuous function [20] if for each x ∈ X and each open set V containing f(x), there is an open set U containing (X, τ) such that f(int(cl(U))) ⊆ int(cl(U)),
7. Almost b-continuous[24] if for each x ∈ X and if for each open set V of (Y, σ) containing f(x), there exists U ∈ BO(X, x) such that f(U) ⊆int(cl(V))
8. Weakly b-continuous[23] if for each x ∈ X and if for each open set V of (Y, σ) containing f(x), there exists b-open set U containing x such that f(U) ⊆ cl(V)
9. R-map[6] if the preimage of every regular open subset of (Y, σ) is a regular open subset of (X, τ).

A topological space (X, τ) is said to be

1. Hausdorff [27] if and only if {x}= ∩{cl(V): x ∈ V ∈ τ } for each x ∈ X.
2. b-T_2 [21], if for each pair of distinct points x and y in (X, τ),there exists U ∈ BO(X,x) and V ∈ BO(X,y) such that U ∩ V = ϕ. i.e., If every two distinct points of (X, τ) can be separated by disjoint b-open sets.
3. Almost compact [19] or quasi-H-closed [23] if every open covering of (X, τ) has a finite subcollection the closures of whose members covers X
4. Lightly compact [5] if every locally finite collection of open subsets of X is finite. .
5. Urysohn [27] for each pair of points x_1 , x_2 ∈ X where $x_1 \neq x_2$, there exists U_1 and U_2 containing x_1 and x_2 respectively such that cl(U_1) ∩ cl(U_2)= ϕ

A nonempty subset A of a topological space (X, τ) is said to be b-closed relative to X [21] if for every cover {V_α : α ∈ I} of A by b-open sets of (X, τ), there exists a finite subset I_0 of I such that A⊆ ∪{ b-cl(V_α) : α ∈ I_0 }.

Lemma 2.1. [24] Let V be a subset of topological space (X, τ). If cl(V) is regular closed, then f^{-1} (cl(V)) is b-closed.

Lemma 2.2. [24] In a topological space (X, τ), every b-continuous function is almost b- continuous.

Lemma 2.3. [21] If A ⊆ X_o ⊆ X and X_o is α-open in topological space (X, τ), then b-cl(A) ∩ X_o = b-cl_{X_o}(A), where b-cl_{X_o}(A) denotes the b-closure of A in the subspace X_o.

Lemma 2.4. [4] Let A and X_o be subsets of a topological space (X, τ).

1. If A ∈ BO(X) and X_o ∈ αO(X), then A ∩ X_o ∈ BO(X_o),
2. If A ∈ BO(X_o) and X_o ∈ αO(X), then A ∈ BO(X).

Lemma 2.5. [18] (X, τ) be a topological space and X_o be an α-open set in X. Then

1. BO(X_o) ={A ⊆ X : A = B ∩ X_o for some b-open set B in X},
2. If U ⊆ X_o and U ∈ BO(X_o), then U ∈ BO(X),
3. If F⊆ X_o and F ∈ BC(X).

Lemma 2.6. [21] For a topological space (X, τ), then the following are equivalent:

1. X is b-regular,
2. For each point x ∈ X and for each open set U of (X, τ) containing x, there exists V ∈ BO(X) such that x ∈ V ⊆ b-cl (V) ⊆ U ,
3. For each subset A of X and each closed set F such that A ∩ F = φ, there exist disjoint U, V ∈ BO (X) such that A ∩ U ≠ φ and F ⊆ V ,

4. For each closed set F of X, F = ⋂ {b-cl(V): F ⊆ V and V ∈ BO (X)}.

Lemma 2.7. [4] In a topological space (X, τ),

1. The intersection of an open set and a b-open set is b-open set,
2. The intersection of an α-open set and a b-open set is b-open set.

3. b-δ-continuous function

Definition 3.1 A function f: (X, τ) → (Y, σ) is said to be b- δ -continuous (briefly b- δ -c) if for each x ∈ X and each open set V of (Y, σ) containing f(x), there exists a b-open set U in (X, τ) containing x such that f(int(bcl(U)))⊆ cl(V).

Theorem 3.2 For a function f: (X, τ) → (Y, σ) the following hold:

1. If f is almost b-continuous, then it is b- δ –continuous,
2. If f is b- δ -continuous, then it is weakly b-continuous.

Proof. 1. Suppose x ∈ X and V is any open set of Y containing f(x). Since f is almost b-continuous, f^{-1}(int(cl(V)) is b-open and since cl(V) is regular closed, f^{-1}(cl(V)) is b-closed by Lemma 2.1. Now set U= f^{-1}(int(cl(V))). Then we have U ∈ BO(X,x) and b-cl(U)⊆ f^{-1}(cl(V)). Hence we have int(b-cl(U))⊆f^{-1}(cl(V)).Therefore we obtain f(int(b-cl(U))) ⊆ cl(V). This shows that f is b- δ -continuous.
2. It is obvious.

Theorem 3.3. Every b-continuous function is b- δ -continuous function.

Proof. Follows from Lemma 2.2. and Theorem 3.2.
The converse of the above theorem need not be true as shown in the following example.

Example 3.4. Let X={a, b, c} and τ = {φ, {a}, {a,b}, {a,c}, X}. Define a function f: (X, τ) → (X, τ) by f(a)=b, f(b)=a and f(c)=a. we have BO(X)= {φ, {a}, {a, b}, {a,c}, X},Then f is b- δ -continuous function but not b-continuous function, since for V ={a} and V ={a, c} there exists no U in BO(X, x) for x=b and x=c such that f(U) ⊆ V.

Remark 3.5. From the above discussions, we have the following diagram. None of the implications is reversible as shown in the following example.

b-continuous function → almost b-continuous function → b- δ -continuous function.

Example 3.6. Let X= {a, b, c, d} and τ = {φ, {b}, {c}, {a,b}, {b,c}, {a,b,c}, X}. Define a function f: (X, τ) → (X, τ) by f(a)=c, f(b)=d, f(c)=d and f(d)=c. Then we have BO(X)= {φ, {b}, {c}, {a,b}, {b,c}, {b,d}, {c,d}, {a,b,c}, {a,b,d}, {b,c,d}, X}. Here f is b- δ -continuous function but not almost b-continuous function, since V={c} there exists no U in BO(X, x) for x = a and x= c such that f(U)⊆ int(cl(V)).

Example 3.7. Let X = Y= {a, b, c} τ ={φ, {a}, {b}, {a,b}, X} and σ ={φ, {a}, {a,b}, X}. Define a function f: (X, τ) → (X, σ) by f(a)=b, f(b)=c, f(c)=a. Then f is almost b-continuous but not b-continuous.

Lemma.3.8. A subset U of a topological space (X, τ) is b- δ -open in (X, τ) if and only if for each x∈ U, there exists a W ∈ BO(X) with x ∈ W such that int(b-cl(W)) ⊆ U.

Proof. Suppose that U is b- δ -open in (X, τ). Then X-U is b-δ-closed. Let x ∈ U. Then x ∉ b- δ -cl(X-U) and so there exists W ∈ BO(X, x) such that int(b-cl(W)) ∩ (X-U)= φ which implies int(b-cl(W)) ⊆ X-(X-U)=U. Thus int(b-cl(W)) ⊆ U. Conversely, assume that U is not a b- δ -open. Then X - U is not b- δ -closed, and so there exists x ∈ b- δ -cl(X-U) such that x ∉ (X - U). Since x ∈ U, by hypothesis, there exists W ∈ BO(X,x) such that int(b-cl(W)) ⊆ U. Thus int(b-cl(W)) ∩(X - U) = φ. This is a contradiction since x ∈ b- δ –cl (X - U).

Theorem 3.9. For a function f: (X, τ) (Y, σ) the following hold:

1. f is b- δ –continuous,
2. b- δ -cl(f^{-1}(int(cl(B))))⊆ f^{-1}(cl(B)) for every subset B of (Y, σ),
3. b- δ -cl(f^{-1}(int(F)))⊆ f^{-1}(F) for every regular closed set F of (Y, σ),
4. b- δ –cl(f^{-1}(int(cl(V))))⊆ f^{-1}(cl(V)) for every V in βO(Y),
5. b- δ -cl(f^{-1}(int(cl(V))))⊆ f^{-1}(cl(V)) for every V in BO(Y),
6. b- δ -cl(f^{-1}(V))⊆ f^{-1}(cl(V)) for every V in PO(Y),
7. b- δ -cl(f^{-1}(V))⊆ f^{-1}(cl(V)) for every open set V of (Y, σ),
8. f^{-1}(V)⊆b- δ -int(f^{-1}(cl(V))) for every open set V of (Y, σ).

Proof. 1. → 2. : Let B be any subset of (Y, σ). Assume that x ∈ X - f^{-1}(cl(B)).Then f(x) ∈ Y - cl(B) and there exists an open set V containing f(x) such that V∩B= φ, hence cl(V)∩ int(cl(B))=φ. Since f is b- δ -continuous, there exists U ∈ BO(X,x) such that f(int(bcl(U)))⊆ cl(V). Therefore we have int(bcl(U))∩f^{-1}(int(cl(B)))=φ and hence x ∈ X -(b- δ -cl(f^{-1}(int(cl(B))))). Thus we obtain b- δ -cl(f^{-1}(int(cl(B))))⊆ f^{-1}(cl(B)).
2.→ 3. : Let F be any regular closed set of (Y, σ). Then we have b-δ-cl(f^{-1}(int(F))) = b-δ-cl(f^{-1}(int(cl(int(F))))) ⊆ f^{1}(cl(int(F))) = f^{-1}(F).
3. → 4. : Let V be any β-open set of (Y, σ). It follows that cl(V) is regular closed. By (3), b- δ -cl(f^{-1}(int(cl(V))))⊆ f^{-1}(cl(V)).
4. → 5. : This is clear, since BO(Y)⊆ βO(Y).
5. → 6. : This is clear, since PO(Y)⊆ BO(Y).
6. → 7. : This is clear, since every open set is preopen.
7.→ 8. : Let V be any open set of (Y, σ). Since Y-cl(V) is open in (Y, σ), we have X-(b- δ -int (f^{-1}(cl(V)))) = b-δ -cl(X- f

^{1}(cl(V))) = b-δ-cl(f^{-1}(Y-cl(V))) ⊆ f^{-1}(cl(Y-cl(V))) = f^{-1}(Y-int(cl(V)))⊆X-f^{-1}(V). Then f^{-1}(V)⊆b- δ -int(f^{-1}(cl(V))).

8.→ 1. : Let x ∈X and V be an open set of (Y, σ) containing f(x). Then x ∈ f^{-1}(V) and, by (8) and Theorem 3.8, there exists U in BO(X, x) such that int(bcl(U))⊆ f^{-1}(cl(V)). Hence f(int((bcl(U)))) ⊆ cl(V) and f is b- δ –continuous.

Theorem 3.10. For a function f: (X, τ) → (X, τ), the following are equivalent:

1. f is b- δ -continuous ,
2. f(b-δ -cl(A))⊆ δ -cl(f(A)) for every subset A of (X, τ),
3. b- δ -cl(f^{-1}(B))⊆ f^{-1}(δ - cl(B)) for every subset B of (Y, σ),
4. b- δ -cl f^{-1}(int (δ-cl(B))))⊆ f^{-1}(δ-cl(B)) for every subset B of (Y, σ).

Proof. 1.→ 2. : Let A be a subset of (X, τ) and x ∈b- δ -cl(A). Let V be any open set of (Y, σ) containing f(x). By (1), there exists U ∈ BO(X, x) such that f(int(bcl(U)))⊆ cl(V). Since x ∈b- δ -cl(A), int(bcl(U))∩A ≠φ and so φ ≠ f(int(bcl(U)))∩ f(A)⊆ cl(V)∩ f(A) so that f(x) ∈ δ -cl(f(A)).

2.→ 3. : Let B be a subset of (Y, σ). we have f(b-δ –cl (f^{-1}(B))) ⊆ δ -cl(f (f^{-1} (B))) ⊆ δ -cl(B) then b- δ -cl(f^{-1}B)) ⊆ f^{-1}(δ -cl(B)).

3.→ 4. : Let B be a subset of (Y, σ). Since δ -cl(B) is closed in (Y, σ), we have b- δ -cl[f^{-1} (int(δ -cl(B))))] ⊆ f^{-1}(cl(int(δ -(cl(B))))) ⊆ f^{-1}(δ -cl(B)).

4.→ 1. : Let V be any open set in (Y, σ). Then V⊆ int(cl(V))=int (δ -cl(V)) and hence b- δ –cl(f^{-1} (V))⊆b- δ -cl(f^{-1}(int(δ -cl(V)))) ⊆ f^{-1}(δ -cl(V))⊆ f^{-1}(cl(V)) and it follows from Theorem 3.9. f is b- δ -continuous .

Theorem 3.11. If f: (X, τ) → (Y, σ) is b- δ -continuous and X_0 is an α-open subset of X, then the restriction f| $_{X0}$: X_0 → Y is b- δ - continuous.

Proof. For any x ∈ X_0 and any open neighborhood V of f(x), there exists U in BO(X, x) such that f(int(b-cl(U))) ⊆ cl(V), since f is b- δ -continuous. Put U_0=U∩X_0, then by Lemma 2.4., U_0 ∈ BO(X_0, x) and by Lemma 2.3., b-cl_{xo}(U_0)⊆ b-cl(U_0). Therefore, we obtain (f|X_0)(int(bcl_{xo}(U_0))= f(int(b-cl_{xo}(U_0))⊆ f(int(b-cl(U_0)) ⊆ f (int(b-cl(U)))⊆ cl(V).

Theorem 3.12. If f: (X, τ) → (Y, σ) is b- δ -continuous and g: (Y, σ) → (Z, η) is δ -continuous, then the composition g o f :(X, τ)→ (Z, η) is b- δ -continuous.

Proof. Let W be an open set in (Z, η).Then b-δ -cl(g o f^{-1}(W)=b-δ –cl(f^{-1} (g $^{-1}$(W))) ⊆ f^{-1}(cl(g $^{-1}$(W))) ⊆ f^{-1}(g^{-1}(cl(W)) ⊆ (g o f)$^{-1}$ (cl(W)) which implies g o f is b- δ -continuous , by Theorem 3.9..

Theorem 3.13. Let f: (X, τ) → (Y, σ) and g :(Y, σ)→ (Z, η) be functions. If g o f:X→ Z is b- δ -continuous, f is surjective and satisfies the condition that f(b-δ -cl(A)) is b- δ -closed for every A⊆ X, then g is b- δ -continuous.

Proof. Let W be an open set in (Z, η). Since g o f:(X, τ)→ (Z, η) is b- δ -continuous, by Theorem 3.9.,b- δ -cl(f^{-1} (g^{-1}(W)))⊆ f^{-1}(g^{-1}(cl(W))) and hence f(b-δ -cl(f^{-1}(g $^{-1}$(W)))⊆ g $^{-1}$(cl(W)). This implies that b- δ -cl[f(f^{-1}(g $^{-1}$(W)))]⊆ g $^{-1}$(W) since b- δ -cl(f(f^{-1}g $^{-1}$(W))))⊆ f(b-δ -cl(f^{-1}(g $^{-1}$(W)))) as f(b-δ -cl(f^{-1}(g $^{-1}$(W)))) is b- δ -closed. Also, since f(b-δ -cl(f^{-1}(g $^{-1}$(W)))) is b- δ -closed, b- δ -cl(f(f^{-1}(g $^{-1}$(W))))⊆ f(b-δ -cl(f^{-1}(g $^{-1}$(W)))). Finally, as f is surjective, it follows that b- δ -cl(g $^{-1}$(W))⊆ g^{-1}(cl(W)). By Theorem 3.9. , g is b- δ -continuous.

Theorem 3.14. If f: (X, τ) → (Y, σ) is a b- δ -continuous , then the following hold:

1. f^{-1}(V) is b- δ -closed for every δ -closed subset V of (Y, σ),
2. f^{-1}(V) is b- δ -open for every δ -open subset V of (Y, σ).

Proof. Let V be a δ -closed set of Y. By Theorem 3.9., we have b- δ -cl(f^{-1} (V)) ⊆ f^{-1} (V). Hence f^{-1}(V) is b- δ -closed. It is obvious that (1) and (2) are equivalent.

Theorem 3.15. If B⊆ A⊆ X and A is an α-open subset of a topological space (X, τ) then b- δ -cl(B)∩ A =b- δ -cl_A(B), where b-δ -cl_A(B) denotes b- δ -closure of B in the subspace A.

Proff. Let x ∈b-δ-cl(B)∩A and let U∈BO(A, x). By Lemma 2.5. , U ∈ BO(X, x) and since x ∈ b-δ-cl(B), int(bcl(U))∩ V≠φ. Thus we have x ∈b-δ-cl_A(B). Hence b-δ-cl(B)∩A⊆ b-δ-cl_A(B).

Conversely, let x ∈b-δ-cl_A(B) and U∈ BO(X, x). By lemma 2.4., U∩ A ∈ BO(A,x). Since x ∈b-δ-cl_A(B)∩int(bcl(U ∩ A))∩ B ≠φ which implies int(bcl(U))∩ B ≠φ. So, x ∈b-δ-cl(B) and x ∈ b-δ-cl(B)∩A. Thus b-δ-cl_A(B)⊆b- δ -cl(B)∩A. Hence b- δ -cl(B)∩A =b-δ -cl_A(B).

Theorem 3.16. Let f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, η) be functions. If g o f is b- δ -continuous and g is a clopen injection, then f is b-δ-continuous.

Proof. Let V ⊆ Y be open. Since g is clopen, g(V) is an open subset of (Z, η). Since g o f is b-δ-continuous, by Theorem 3.9., b-δ-cl(g o f)$^{-1}$ (g(V)) ⊆ (g o f)$^{-1}$ (cl(g(V))) = f^{-1} (g $^{-1}$(cl(g(V)))). Furthermore, since g is closed and injective, b-δ-cl(f^{-1}(V)) = b-δ-cl(f^{-1}(g^{-1}g(V))) ⊆ f^{-1}(g $^{-1}$(cl(g(V))))⊆ f^{-1}(cl(g^{-1}(g(V)))) = f^{-1}(cl(V)). This shows that f is b-δ-continuous.

Theorem 3.17. Let f: (X, τ) →(Y, σ) be b-δ-continuous surjection. If (X, τ) is b-closed, then (Y, σ) is quasi-H-closed.

Proof. Let {V_α }$_{\alpha \in I}$ be an open cover of (Y, σ). If x ∈ X, then f(x) ∈ $V_{\alpha(x)}$ for some α(x) ∈I. Since f is b-δ-continuous, there exists a b-open set U_x such that f(int(b-cl(U_x))) ⊆

$cl(V_{\alpha(x)})$. Thus $\{U_x\}_{x \in X}$ is a b-open cover of X. Since X is b-closed, there exist $x_1, x_2, .. x_n \in X$ such that $X \subseteq U\{b\text{-}cl(U_{xi}) : x_i \in X, i=1,2,3,....n\}$. So we obtain $f(X) \subseteq f(U\{b\text{-}cl(U_{xi}) : x_i \in X, i=1,2...n\}) \subseteq U\{cl(V_{\alpha(xi)}) : x_i \in X, i=1,2...n\}$. Since f is surjective, (Y, σ) is quasi-H-closed.

Theorem 3.18. If f: (X, τ) → (Y, σ) is continuous, and then f is b- δ -continuous.

Proof. Let V be an open subset of (Y, σ). Since $f^{-1}(V)$ is open, it follows that $b\text{-}\delta\text{-}cl(f^{-1}(V)) \subseteq cl(f^{-1}(V))$ and since f is continuous $cl(f^{-1}(V)) \subseteq f^{-1}(cl(V))$. Therefore, we obtain that $b\text{-}\delta\text{-}cl(f^{-1}(V)) \subseteq f^{-1}(cl(V))$. This shows that f is b-δ-continuous by Theorem 3.9..

Definition 3.19. A function f: (X, τ) → (Y, σ) is called contra b-δ–continuous if $f^{-1}(V)$ is b-δ-closed set in X for every open set V in (Y, σ).

Theorem 3.20. If f: (X, τ) → (Y, σ) is contra b-δ -continuous , then f is b- δ -continuous .

Proof. Let V be any open set in (Y, σ). Then, $f^{-1}(V)$ is b-δ-closed set. Thus $b\text{-}\delta\text{-}cl(f^{-1}(V)) = f^{-1}(V) \subseteq f^{-1}(cl(V))$. Hence by Theorem 3.9. , f is b- δ -continuous .

Theorem 3.21. Let (X, τ) be a b-regular space. Then f: (X, τ) → (Y, σ) is b- δ -continuous if and only if f is weakly b-continuous.

Proof. Necessity is obvious.
Sufficiency: Suppose that f is weakly b-continuous. Let x ∈ X and V be any open set of (Y, σ) containing f(x). Then there exists U ∈ BO(X, x) such that $f(U) \subseteq cl(V)$. Since (X, τ) is b-regular, there exists W ∈ BO(X, x) such that $x \in W \subseteq bcl(W) \subseteq U$ by Lemma 2.6.. Therefore we obtain $f(bcl(W)) \subseteq cl(V)$. Hence $f(int(bcl(W))) \subseteq cl(V)$. This shows f is b- δ -continuous.

Theorem 3.22. If f :(X, τ) → (Y, σ) is a b- δ -continuous injection and (Y, σ) is Urysohn, then (X, τ) is b-T_2.

Proff. Let $x_1, x_2 \in (X, \tau)$ and $x_1 \neq x_2$. Since f is injective and since (Y, σ) is Urysohn, $f(x_1) \neq f(x_2)$ and there exists open sets V_1 and V_2 containing $f(x_1)$ and $f(x_2)$ respectively such that $cl(V_1) \cap cl(V_2) = \varphi$. Since f is b-δ-continuous , there exist b-open sets U_1 and U_2 such that $x_1 \in U_1$ and $x_2 \in U_2$ and $f(int(bcl(U_1))) \subseteq cl(V_1)$ and $f(int(bcl(U_2))) \subseteq cl(V_2)$. Therefore, we obtain disjoint b-open sets U_1 and U_2 containing x_1 and x_2 respectively. Thus X is b-T_2.

Theorem 3.23. Let f,g :(X, τ)→ (Y, σ) be functions and (Y, σ) be a Hausdorff space. If f is b-δ-continuous and g is an R-map, then the set A={x ∈ X : f(x)=g(x)} is b-closed in X.

Proff. Let x ∉ A, so that f(x) ≠ g(x). Since (Y, σ) is Hausdorff, there exist open sets V_1 and V_2 in (Y, σ) such that $f(x) \in V_1$, $g(x) \in V_2$ and $V_1 \cap V_2 = \varphi$. Hence $cl(V_1 \cap int(cl(V_2))) = \varphi$. Since f is b-δ-continuous, there exists G ∈ BO(X) containing x such that $f(int(b\text{-}cl(G))) \subseteq cl(V_1)$. Since g is an R-map, $g^{-1}(int(cl(V_2)))$ is a regular open subset of X and $x \in g^{-1}(int(cl(V_2)))$. Put $U = G \cap g^{-1}(int(cl(V_2)))$. By Lemma 2.7., x ∈ U in BO(X) and U∩A=φ, which implies x ∉ b-cl(A). Thus $b\text{-}cl(A) \subseteq A$. But $A \subseteq b\text{-}cl(A)$. Hence A= b-cl(A) which implies A is b-closed in (X, τ).

Theorem 3.24. If a function f: (X, τ) → (Y, σ) is b- δ -continuous and K is b-closed relative to (X, τ), then f(K) is quasi H-closed relative to (Y, σ).

Proff. Suppose that f: (X, τ) → (Y, σ) is b-δ-continuous and K is b-closed relative to X. Let $\{V_\alpha : \alpha \in I\}$ be a cover of f(K) by open sets of (Y, σ). For each point x ∈ K, there exists α(x) ∈ I such that $f(x) \in V_{\alpha(x)}$. Since f is b-δ-continuous, there exists a $U_x \in BO(X, x)$ such that $f(int(b\text{-}cl(Ux))) \subseteq cl(V_{\alpha(x)})$. The family $\{U_x : x \in K\}$ is a cover of K by a b-open sets of X and hence there exists a finite subset K_0 of K such that $K \subseteq U_{x \in Ko}\, b\text{-}cl(U_x)$, since K is b-closed relative to X. Therefore, we obtain $f(K) \subseteq U_{x \in Ko}\, cl(V_{\alpha(x)})$. This shows that f(K) is quasi H-closed relative to (Y, σ).

Theorem 3.25. Let f: (X, τ) → (Y, σ) be a b- δ -continuous surjection. Then the following hold:

1. If (X, τ)is b-closed, then (Y, σ) is almost compact,
2. If (X, τ) is countably b-closed, then (Y, σ) is lightly compact.

Proff.1. Let $\{V_\alpha : \alpha \in I\}$ be a cover of (Y, σ) by open sets of (Y, σ). For each point x ∈ X, there exists α(x) ∈ I such that $f(x) \in V_{\alpha(x)}$. Since f is b-δ-continuous , there exists a b-open set U_x of (X, τ) containing x such that $f(int(b\text{-}cl(U_x))) \subseteq cl(V_{\alpha x}))$. The family $\{U_x : x \in K\}$ is a cover of X by b-open sets of X and hence there exists a finite subset X_0 of (X, τ) such that $X \subseteq U_{x \in Xo}(b\text{-}cl(U_x))$. Therefore, we obtain $Y = f(X) \subseteq U_{x \in Xo}\, (cl((V_{\alpha(x)}))$. This shows that (Y, σ) is almost compact.
2.Similar to 1.

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