IJETST- Vol.||02||Issue||07||Pages 2890-2895||July||ISSN 2348-9480 2015

Open access Journal International Journal of Emerging Trends in Science and Technology

Some Important Properties of Intuitionistic Fuzzy Soft Sequentially Compact & Totally Bounded *(IFS)*˜ Spaces

Authors

A.Sreedevi[1], Dr. N. Ravi Shankar[2]

[1]CMR Engg College, Kandlakoya, Hyderabad, India

Email: *Sreedevi.agastyaraju@gmail.com*

[2]GIS, GITAM University, Vizag, India

Email: drravi68@gmail.com

Abstract

The purpose of this paper is to define (IFS)˜(intuitionistic fuzzy soft) sequential compact space and to investigate some important theorems on it. In this view we define (IFS)˜soft point, (IFS)˜ ἕ net, (IFS)˜totally bounded space and investigate their properties. In continuation we define Lebesque (IFS)˜number, uniformly continuous (IFS)˜mapping and study few theorems.

Keywords: *(IFS)˜Real number, (IFS)˜point, (IFS)˜sequentially compact space, (IFS)˜ ἕ- net ,(IFS)˜totally bounded space, uniformly continuous (IFS)˜mapping.*

1. Introduction

The word, soft set is introduced by Molodtsov[1] as an innovative mathematical tool to handle uncertainties which occur in the developments and progress of Economics, Social Science, Environment, Engineering, Medical Sciences etc. Soft set theory is being applied in many areas such as game theory, Real Analysis, Functional Analysis and in Topology also. P.K. Maji, A.R. Roy[4] have introduced Fuzzy soft sets and Tanay, Bekir, and M. Burc Kandemir[3] developed topological structures in Fuzzy soft theory.Many researchers have been improving this theory in different areas like information systems, images, forecasting and more in decision making etc.In continuation Jiang, Yuncheng and Qimai Chen [4] have worked on an adjustable approach to intuitionistic fuzzy soft sets. Later Gunduz, Cigdem and Sadi Bayramov[5] studied Intuioinistic fuzzy soft theory and introduced Intuioinistic fuzzy soft modules from which Bayramov and Cigdem Gunduz [6] have investigated different basic properties and developed many theorems on topology.

In our paper we try to give an extension of the concept of Topological spaces in Intuioinistic fuzzy soft theory.We define *(IFS)*˜Sequentially compact space and examine some important theorems on *(IFS)*˜Sequentially compact space.We also introduce *(IFS)*˜ἕ- net ,totally bounded *(IFS)*˜metric space and we will study properties of this space.Finally we define Lebesgue *(IFS)*˜ number and uniformly continuous *(IFS)*˜ mapping and investigate some important theorems.

2. Preliminaries

Throughout this paper, X means an initial universe, E is the set of all parameters for X. These parameters may be attributes, characteristics or properties of some objects.

Definition 2.1. [(1)] A pair $[\widetilde{f,A}]$ is called a soft set over X, where f is a mapping given by $f : A \to P(X)$. i.e The soft set is a parameterized family of subsets of the set X.

Definition 2.2 [9] An Intuitionistic Fuzzy set A over the universe X can be defined as follows

$A = \{(x, \tilde{\mu} A (x), \tilde{\nu} A (x)) : x \in X \}$

where $\tilde{\mu} A : X \to [0, 1]$, $\tilde{\tilde{\nu}} A : X \to [0, 1]$ with the property $0 \le \tilde{\mu} A (x)+\tilde{\tilde{\nu}} A (x) \le 1$, $\forall x \in X$. The values $\tilde{\mu} A (x)$ and $\tilde{\tilde{\nu}} A (x)$ represent the degree of membership and nonmembership of x to A respectively.

Definition 2.3 [5] Let IFS^S denote the collection of all Intuitionistic Fuzzy subsets of X . Let A⊆E. A pair $[(f,A)^{\sim}]$ is called an *(IFS)*˜set over X where f is a mapping given by $f : A \to IFS^S$. The set of all *(IFS)*˜sets over X with parameters from E is called an *(IFS)*˜class and it is denoted by $IFS[\tilde{X}_E]$.

Definition 2.4 [5]: Let $[(f,A)^{\sim}]$ and $[(g,B)^{\sim}]$ be two *(IFS)*˜sets over X . Then $[(f,A)^{\sim}] \cup [(g,B)^{\sim}] = [(h, C)^{\sim}]$ where $C = A \cup B$ and $\forall e \in C$,

$h(e) = f(e)$ if $e \in A - B$,

$h(e) = f(e)$, if $e \in B - A$,

$h(e) = f(e) \cup g(e)$ if $e \in A \cap B$.

Definition 2.5 [5] $[(f,A)^{\sim}] \cap [(g,B)^{\sim}] = [(h, C)^{\sim}]$ where $C = A \cap B$ and $\forall e \in C$, $h(e) = f(e) \cap g(e)$.

Definition 2.6 [5] $[(f,A)^{\sim}] \subseteq [(g,B)^{\sim}]$ where i) $A \subseteq B$, ii) for all $e \in A$, $f(e) \subseteq g(e)$.

Definition 2.7 [5] The complement of an *(IFS)*˜set $[(f,A)^{\sim}]$ is denoted by $[(f,A)^{\sim}]^c$ and is defined by $[(f,A)^{\sim}]^c = (f^c , A)$, where $f^c : A \to IF^U$ is a mapping given by $f^c (e) = [f (e)]^c$ for all $e \in$

A. Thus if f (e) = (x, $\tilde{\mu}_{F(e)}(x)$, $\tilde{\tilde{\nu}}_{F(e)}(x)$) : x ∈ X, then ∀e ∈ A, f^c (e) = (f (e))^c = {x, $\tilde{\tilde{\nu}}_{F(e)}(x)$, $\tilde{\mu}_{F(e)}(x)$) : x ∈ X }.

Definition 2.8 [5]: A soft set [(f,A)˜] over X is said to be absolute *(IFS)*˜set denoted by $\tilde{X}_A$ if ∀e ∈ A, f (e) is the absolute intuitionistic fuzzy set ĩ of U where ĩ(x) = 1, ∀ x ∈ U.

Definition 2.9 [5]: A soft set [(f,A)˜] over X is said to be null *(IFS)*˜set denoted by $\tilde{\varphi}_A$ if ∀e ∈ A, f (e) is the null intuitionistic fuzzy set õ of U where õ (x) = 0, ∀ x ∈ U.

Definition 2.10 [7] Let τX ⊆ *IFS*($\tilde{X}_E$), then τX is said to be a *(IFS)*˜topology on X if the following conditions hold.

i. $\tilde{\varphi}_A$, $\tilde{X}_A$ belong to τX .
ii. The union of any number of *(IFS)*˜sets in τX belongs to τX .
iii. The intersection of any two *(IFS)*˜sets in τX belongs to τX .

Note: τX is called a *(IFS)*˜topology over X and the ordered pair ($\tilde{X}_E$, τX) is called a *(IFS)*˜topological space over X. The members of τX are said to be *(IFS)*˜open sets in X. A *(IFS)*˜set [(f, E)˜] over X is said to be a *(IFS)*˜closed set in X, if its complement [(f, E)˜] ^{c} belongs to τ .

Definition 2.11 [7] Let ($\tilde{X}_E$, τX) be a *(IFS)*˜topological space over $\tilde{X}_E$ and $\tilde{Y}$ be a non-empty subset of $\tilde{X}$. Then τY = {($\tilde{Y}_F$, E) : [(f, E)˜] ∈ T }is said to be the *(IFS)*˜topology on Y and ($\tilde{Y}_F$, τY) is called a *(IFS)*˜subspace of ($\tilde{X}_E$, τ). Where ($\tilde{Y}_F$, E) = $\tilde{Y}_F$ ∩ [(f, E)˜] . Here τY is, in fact, a *(IFS)*˜topology on Y.

Definition 2.12 [7] A family {[(fi, A)˜] }$_{i\in\Delta}$ *(IFS)*˜sets is said to be a cover of a *(IFS)*˜set [(f,A)˜] if

[(f,A)˜] ⊆̃ Ũ [(fi, A)˜] . It is a *(IFS)*˜open cover if each member of {[(fi, A)˜] }$_{i\in\Delta}$ is a *(IFS)*˜open set.A finite collection of [(f,A)˜] is a subfamily of {[(fi, A)˜] }$_{i\in\Delta}$ if is also a cover.

Definition 2.13 [7] Let ($\tilde{X}_E$, τ) be *(IFS)*˜topological space and [(f,A)˜] ∈̃ *IFS*[$\tilde{X}_E$].*(IFS)*˜set [(f,A)˜] is called *(IFS)*˜compact if each *(IFS)*˜open cover of [(f,A)˜] has a finite subcover.Also *(IFS)*˜topological space ($\tilde{X}_E$, τ) is called *(IFS)*˜compact space if each *(IFS)*˜open cover of $\tilde{X}_E$ has a finite subcover.

Intuitionistic Fuzzy Soft Real numbers

Definition 3.1 An *(IFS)*˜set A = {(x, $\tilde{\mu}$A (x), $\tilde{\tilde{\nu}}$A (x)) : x ∈ X } over the universe X is said to bounded if there is a real number r where 0 < r < 1 such that $\tilde{\mu}$A (x) < 1- r and $\tilde{\tilde{\nu}}$A (x) < r.

Definition 3.2. Let **R** be the set of all real numbers and IF^R be the collection of all non-empty bounded IF subsets of **R** and E taken as a set of parameters.Then a mapping f: E → IF^R is called an *(IFS)*˜ real set.It is denoted by [f, E]^R.If specifically [f, E]^R is a singleton *(IFS)*˜set, then identifying [f, E]^R with the corresponding *(IFS)*˜element, it will be called an *(IFS)*˜real number and denoted by $\check{r}$,$\check{s}$,$\check{t}$ etc.

Note: (1) $\check{0}$ = {(0,0,1) : 0 ∈ R}
(2) $\check{1}$ = {(1,1,0) : 1 ∈ R}
(3) $\check{r}$ = {(r, $\mu_A(r), \vartheta_A(r): r \in R$}
(4) $\check{s}$ = {(s, $\mu_A(s), \vartheta_A(s): s \in R$} where A ⊆ E.etc.

Definition 3.3 For two *(IFS)*˜real numbers

(1). $\check{r}$ ≧̃ $\check{s}$ if $\check{r}$ (e) ≥ $\check{s}(e)$ ∀ e ∈ E.
(2). $\check{r}$ ≦̃ $\check{s}$ if $\check{r}$ (e) ≤ $\check{s}(e)$ ∀ e ∈ E.
(3). $\check{r}$ >̃ $\check{s}$ if $\check{r}$ (e) > $\check{s}(e)$ ∀ e ∈ E.
(4). $\check{r}$ <̃ $\check{s}$ if $\check{r}$ (e) < $\check{s}(e)$ ∀ e ∈ E.

Definition 3.4 A *(IFS)*˜set [(f,A)˜] is said to be a *(IFS)*˜point, denoted by $e_F x$, if there is one e ∈ E, such that f(e) = {x} for some x ∈ X and f(e') = φ, ∀ e' ∈ E-{e}.

Definition 3.5 Two *(IFS)*˜points $e_F x$, $e'_F y$ corresponding to the *(IFS)*˜sets [(f,A)˜] and [(g,E)˜] respectively(i.e. F(e) = {x}; G(e')={y}), are said to be equal if e = e' and F(e) = G(e'). i.e. x = y. Thus $e_F x \neq e'_F y$ iff x ≠ y or e ≠ e'.

Definition 3.6 A *(IFS)*˜point $e_F x$ is said to be in a *(IFS)*˜set [g, A],denoted by $e_F x$ ∈ [g, A] if for the element e ∈ A; f(e) ≤ g(e).

Definition 3.7 Let ($\tilde{X}_E$, τX)be a *(IFS)*˜topological space over X.Then *(IFS)*˜interior of [(f,A)˜] denoted by [(f,A)˜]^0 is defined as the union of all *(IFS)*˜opensets contained in [(f,A)˜].

Definition 3.8 Let ($\tilde{X}_E$, τX)be a *(IFS)*˜topological space over X.Then *(IFS)*˜closure of [(f,A)˜], denoted by $\overline{(F,E)}$, is defined as the intersection of all *(IFS)*˜closed super sets of [(f,A)˜].

Definition 3.9 Let ($\tilde{X}_E$, τX)be a *(IFS)*˜topological space and [(f,A)˜] be a *(IFS)*˜set in ($\tilde{X}_E$, τX). A *(IFS)*˜set [(g,B)˜] in ($\tilde{X}_E$, τX) is said to be a *(IFS)*˜neighbourhood of [(f,A)˜] if there exists a *(IFS)*˜open set [(h, C)˜] ∈ τX such that [(f,A)˜] ⊆ [(h, C)˜] ⊆ [(g,B)˜].

Definition 3.10 Let ($\tilde{X}_E$, τX) and ($\tilde{Y}_F$, τY) be two *(IFS)*˜topological spaces, $\tilde{f}$: ($\tilde{X}_E$, τX)⟶ ($\tilde{Y}_F$, τY) be a mapping. For each *(IFS)*˜neighbourhood [h, E] of ($\tilde{f}$ $(x)_e$,E), if there exists a *(IFS)*˜neighbourhood [(f,A)˜] of (x_e,E) such that $\tilde{f}$ ([(f,A)˜]) ⊆̃[h, E], then $\tilde{f}$ is said to be *(IFS)*˜continuous mapping at (x_e,E).

Let $\tilde{X}_E$ be the *(IFS)*˜absolute set i.e., f(e)=1 ∀ $e \in E$,Where [(f,A)˜] = $\check{1}$ and *(IFSP)*˜($\tilde{X}$) be the collection of all *(IFS)*˜points of $\tilde{X}_E$ and IFR(E) denote the set of all non negative *(IFS)*˜real numbers.

Definition 3.11 A mapping $\tilde{d}$: *(IFSP)*˜($\tilde{X}_E$) X *(IFSP)*˜($\tilde{X}_E$) → IFR(E), is said to be *(IFS)*˜metric on the *(IFS)*˜set $\tilde{X}_E$ if $\tilde{d}$ satisfies the following conditions.

(i). $\tilde{d}(e_F x \, , e_F y) \supseteqq \tilde{0}$,
(ii). $\tilde{d}(e_F x \, , e_F y) = \tilde{0}$ if and only if $e_F x = e_F y$.
(iii). $\tilde{d}(e_F x \, , e_F y) = \tilde{d}(e_F y \, , e_F x)$
(iv). $\tilde{d}(e_F x \, , e_F y) \leqq \tilde{d}(e_F x \, , e_F z) + \tilde{d}(e_F z \, , e_F y)$ for all $e_F x$, $e_F y$, $e_F z \tilde{\in}$ *(IFSP)*˜$(\tilde{X}_E)$.

The *(IFS)*˜set *(IFSP)*˜$(\tilde{X}_E)$ with the *(IFS)*˜metric $\tilde{d}$ on *(IFSP)*˜$(\tilde{X}_E)$ is called a *(IFS)*˜metric space and denoted by (*(IFSP)*˜$(\tilde{X}_E)$, $\tilde{d}$, E).

Definition 3.12 Let *(IFSP)*˜$(\tilde{X}_E)$ be a *(IFS)*˜metric space and $\breve{\varepsilon}$ be a non negative *(IFS)*˜real number.

Then *(IFSB)*˜$(e_F x, \breve{\varepsilon})$= { $e'_F y \tilde{\in}$ *(IFSP)*˜$(\tilde{X}_E)$; $\tilde{d}(e_F x \, , e'_F y) \lesssim \breve{\varepsilon}$ } ⊂ *(IFSP)*˜$(\tilde{X}_E)$ is called the *(IFS)*˜open ball with center $e_F x$ and radius $\breve{\varepsilon}$ and

(IFSB)˜$(e_F x, \breve{\varepsilon})$= {($e_F x \tilde{\in}$ *(IFSP)*˜$(\tilde{X}_E)$; $\tilde{d}(e_F x \, , e'_F y) \leqq \breve{\varepsilon}$} ⊂ *(IFSP)*˜$(\tilde{X}_E)$ is called the *(IFS)*˜closed ball with center center $e_F x$ and radius $\breve{\varepsilon}$.

Definition 3.13 Let $\{e_F x_{\alpha,n}\}_n$ be a sequence of *(IFS)*˜points in a *(IFS)*˜metric space (*(IFSP)*˜$(\tilde{X}_E)$, $\tilde{d}$, E).The sequence $\{e_F x_{\alpha,n}\}_n$ is said to be convergent in (*(IFSP)*˜$(\tilde{X}_E)$, $\tilde{d}$, E) if there is a *(IFS)*˜point $e_F y_\omega \tilde{\in}$ *(IFSP)*˜$(\tilde{X}_E)$ such that

$\tilde{d}(\, e_F x_{\alpha,n}, e_F y_\omega) \to \tilde{0}$ as n → ∞.
This means for every $\breve{\varepsilon} \gtrsim \tilde{0}$, chosen arbitrarily, there exists anatural number N = N($\breve{\varepsilon}$) , such that $\tilde{0} \leqq \tilde{d}(\, e_F x_{\alpha,n}, e_F y_\omega) \lesssim \breve{\varepsilon}$, whenever n > N.
Theorem3.14 Limit of a sequence in a *(IFS)*˜metric space , if exists is unique.

Definition 3.15 (Cauchy Sequence). A sequence $\{e_F x_{\alpha,n}\}_n$ of *(IFS)*˜points in (*(IFSP)*˜$(\tilde{X}_E)$, $\tilde{d}$, E) is considered as a cauchy sequence in *(IFSP)*˜$(\tilde{X}_E)$ if corresponding to every $\breve{\varepsilon} \gtrsim \tilde{0}$, there exists n ∈ N such that $\tilde{d}(e_F x_{\alpha,i} \, , e_F x_{\alpha,j}) \leqq \breve{\varepsilon}$, ∀ i, j ≥ m, i.e., $\tilde{d}(e_F x_{\alpha,i} \, , e_F x_{\alpha,j}) \to \tilde{0}$ as i,j → ∞.

Definition 3.16 Let (*(IFSP)*˜$(\tilde{X}_E)$, $\tilde{d}$, E) be a *(IFS)*˜metric space. (*(IFSP)*˜$(\tilde{X}_E)$, $\tilde{d}$, E) is called *(IFS)*˜Sequentially compact space if every *(IFS)*˜sequence has a *(IFS)*˜sequence that converges in *(IFSP)*˜$(\tilde{X}_E)$.

i.e., Suppose $\{e_F x_{\alpha,n}\}_n$ is a *(IFS)*˜sequence in *(IFSP)*˜$(\tilde{X}_E)$ then there exists a subsequence $\{e_F x_{\alpha,n,k}\}_n$ from $\{e_F x_{\alpha,n}\}_n$ such that $\lim_{n\to\infty} e_F x_{\alpha,n,k} = e_F x$.

Theorem 3.17 A *(IFS)*˜metric space (*(IFSP)*˜$(\tilde{X}_E)$, $\tilde{d}$, E) is *(IFS)*˜sequential compact sapce if and only if every infinite *(IFS)*˜subset of *(IFSP)*˜$(\tilde{X}_E)$, has a limit point.

Proof. Let (*(IFSP)*˜$(\tilde{X}_E)$, $\tilde{d}$, E) be a *(IFS)*˜metric space.

Let *(IFSP)*˜$(\tilde{X}_E)$ be a *(IFS)*˜sequential compact space.

Now we show that every infinite *(IFS)*˜subset of *(IFSP)*˜$(\tilde{X}_E)$, has a limit point.

Let *(IFSP)*˜$(\tilde{A})$ be an infinite subset of *(IFSP)*˜$(\tilde{X}_E)$.

Since *(IFSP)*˜$(\tilde{A})$ is infinite,a sequence $\{e_F x_{\alpha,n}\}_n$ of distinct points can be extracted from *(IFSP)*˜$(\tilde{A})$.By definition of *(IFS)*˜sequential compact space has a convergent subsequence.

i.e., There exists $\{e_F x_{\alpha,n,k}\}$ such that $\lim_{n\to\infty} e_F x_{\alpha,n,k} = e_F x$.
And clearly $e_F x$ is a limit point of $\{e_F x_{\alpha,n}\}_n$.
Therefore is a limit point of the set *(IFSP)*˜$(\tilde{A})$ of *(IFS)*˜points of $\{e_F x_{\alpha,n}\}_n$.

Conversely suppose that every infinite IFsoft subset of *(IFSP)*˜$(\tilde{X}_E)$, has a limit point.

Now we show that (*(IFSP)*˜$(\tilde{X}_E)$, $\tilde{d}$, E) is *(IFS)*˜sequential compact sapce.

Consider an arbitrary *(IFS)*˜sequence $\{e_F x_{\alpha,n}\}_n$ in *(IFSP)*˜$(\tilde{X}_E)$.

If $\{e_F x_{\alpha,n}\}_n$ has a point which is repeated infinite times, then possesses a constant convergent sub sequence .
If $\{e_F x_{\alpha,n}\}_n$ has no *(IFS)*˜point which is repeated then the set *(IFSP)*˜$(\tilde{A})$ of infinite *(IFS)*˜points of this sequence is infinite.

By hypothesis *(IFSP)*˜$(\tilde{A})$ contains a limit point in *(IFSP)*˜$(\tilde{X}_E)$, say $e_F x$.

And easily we can find a subsequence of $\{e_F x_{\alpha,n}\}_n$ which converges to $e_F x$.
Hence (*(IFSP)*˜$(\tilde{X}_E)$, $\tilde{d}$, E) is *(IFS)*˜sequential compact space.

Theorem 3.18 Let (*(IFSP)*˜$(\tilde{X}_E)$, $\tilde{d}$, E) be *(IFS)*˜compact metric space.Then every infinite subset of *(IFSP)*˜$(\tilde{X}_E)$ has a limit point.

Proof. Suppose (*(IFSP)*˜$(\tilde{X}_E)$, $\tilde{d}$, E) is a *(IFS)*˜compact metric space.

Let *(IFSP)*˜$(\tilde{A})$ be an infinite subset of *(IFSP)*˜$(\tilde{X}_E)$.

In a contrary way suppose that *(IFSP)*˜$(\tilde{A})$ has no limit point in .

Let $e_F x \tilde{\in}$ *(IFSP)*˜$(\tilde{X}_E)$.

Then $e_F x$ is not a limit point of *(IFSP)*˜$(\tilde{A})$.

So there a *(IFS)*˜neighbourhood *(IFSB)*˜$(e_F x, \breve{\varepsilon})$ such that *(IFSB)*˜$(e_F x, \breve{\varepsilon}) \tilde{\nsubseteq}$ *(IFSP)*˜$(\tilde{A})$

This can be done for each *(IFS)*˜point $e_F x$ in *(IFSP)*˜$(\tilde{X}_E)$.

Consider the class { *(IFSB)*˜$(e_F x, \breve{\varepsilon})$/ $e_F x \tilde{\in}$ *(IFSP)*˜$(\tilde{X}_E)$}.

Trivially *(IFSP)*˜$(\tilde{X}_E)$ = $\tilde{\cup}$ *(IFSB)*˜$(e_F x, \breve{\varepsilon})$.

Since each is *(IFS)*˜open set, { *(IFSB)*˜$(e_F x, \check{\varepsilon})$/ $e_F x$ $\tilde{\in}$ *(IFSP)*˜$(\tilde{X}_E)$} forms an *(IFS)*˜open cover for *(IFSP)*˜$(\tilde{X}_E)$ and this open cover contains a finite subcover (Since *(IFSP)*˜$(\tilde{X}_E)$ is a *(IFS)*˜compact space).

∴ *(IFSP)*˜$(\tilde{X}_E)$ = *(IFSB)*˜$(e_F x1, \check{\varepsilon})$ $\tilde{\cup}$ *(IFSB)*˜$(e_F x2, \check{\varepsilon})$ $\tilde{\cup}$ *(IFSB)*˜$(e_F x3, \check{\varepsilon})$... $\tilde{\cup}$ *(IFSB)*˜$(e_F xk, \check{\varepsilon})$, finite union.

But *(IFSP)*˜$(\tilde{A})$ $\tilde{\subseteq}$ *(IFSP)*˜$(\tilde{X}_E)$.

Which is not possible as *(IFSP)*˜$(\tilde{A})$ is infinite.

This is a contradiction.
Hence the theorem is proved.

Definiton 3.19 Suppose (*(IFSP)*˜$(\tilde{X}_E)$, $\tilde{d}$, E) is a *(IFS)*˜metric space and *(IFSP)*˜$(\tilde{A})$ $\tilde{\subseteq}$ *(IFSP)*˜$(\tilde{X}_E)$.Then diameter of *(IFSP)*˜$(\tilde{A})$ is denoted by Diam(*(IFSP)*˜$(\tilde{A})$) or D(*(IFSP)*˜$(\tilde{A})$) and is defined as Diam(*(IFSP)*˜$(\tilde{A})$) = Sup { $\tilde{d}(e_F x$, $e_F y$): $e_F x$, , $e_F y$ $\tilde{\in}$ *(IFSP)*˜$(\tilde{A})$}.

Definition 3.20 Suppose (*(IFSP)*˜$(\tilde{X}_E)$, $\tilde{d}$, E) is a *(IFS)*˜topological space and

Ω = { *(IFSP)*˜$(\widetilde{B_n})$: n ∈ Δ} be a *(IFS)*˜open cover for *(IFSP)*˜$(\tilde{X}_E)$.Then a *(IFS)*˜real number $\check{r}$ > 0 is called a Leabegue's $\widetilde{IFS}$ number (*LIFSN* in short) if every subset *(IFSP)*˜$(\tilde{A})$ of *(IFSP)*˜$(\tilde{X}_E)$ with diameter less than $\check{r}$ and *(IFSP)*˜$(\tilde{A})$ is contained in atleast one member of the *(IFS)*˜open cover Ω.

Definition 3.21 A subset *(IFSP)*˜$(\tilde{A})$ of *(IFSP)*˜$(\tilde{X}_E)$ is said to be bounded if its diameter is finite.

In particular a *(IFS)*˜metric space (*(IFSP)*˜$(\tilde{X}_E)$, $\tilde{d}$, E) is bounded if Diam(*(IFSP)*˜$(\tilde{X}_E)$) <∞.

Lemma 3.22 In a *(IFS)*˜Sequentially compact space every open cover has a *(IFS)*˜Lebesgue's number.

Proof. Let (*(IFSP)*˜$(\tilde{X}_E)$, $\tilde{d}$, E) be a *(IFS)*˜Sequentially compact space.

Let Ω = { *(IFSP)*˜$(\widetilde{B_n})$: n ∈ Δ} be a *(IFS)*˜open cover for .

We show that this open cover has a *IFS*L number.

In a contrary way suppose that Ω has no *IFS*L number.

Then ∃ *(IFSP)*˜$(\widetilde{C_n})$ from *(IFSP)*˜$(\tilde{X}_E)$ such that Diam(*(IFSP)*˜$(\widetilde{C_n})$) < $\frac{1}{n}$ and *(IFSP)*˜$(\widetilde{C_n})$ $\tilde{\nsubseteq}$ *(IFSP)*˜$(\widetilde{B_n})$ for any n.........(i)

Now choose *(IFS)*˜point $e_F x_n$ from *(IFSP)*˜$(\widetilde{C_n})$ and construct the sequence {$e_F x_n$ }.

Since *(IFSP)*˜$(\tilde{X}_E)$ is *(IFS)*˜sequential compact, the above sequence has a convergent subsequence,say {$e_F x_{n,k}$ }.

⇒ {$e_F x_{n,k}$ } → $e_F x$ as n → ∞.

∴ $e_F x$ belongs to atleast one member of Ω, say *(IFSP)*˜$(\widetilde{B_{n0}})$.Since *(IFSP)*˜$(\widetilde{B_{n0}})$ is IS soft open set there is a *(IFS)*˜open ball *(IFSB)*˜$(e_F x, \check{\varepsilon})$ with center at $e_F x$ such that *(IFSB)*˜$(e_F x, \check{\varepsilon})$ $\tilde{\subseteq}$ *(IFSP)*˜$(\widetilde{B_{n0}})$......(ii)

Now we consider *(IFSB)*˜$(e_F x, \frac{\check{\varepsilon}}{2})$, a concentric circle of *(IFSB)*˜$(e_F x, \check{\varepsilon})$.

Clearly *(IFSB)*˜$(e_F x, \frac{\check{\varepsilon}}{2})$ $\tilde{\subseteq}$ *(IFSB)*˜$(e_F x, \check{\varepsilon})$.......... (iii)

∵ {$e_F x_{n,k}$ } converges to $e_F x$, ∃ n0 ∈ N such that nk ≥ n0, $e_F x_{n,k}$ $\tilde{\in}$ *(IFSB)*˜$(e_F x, \frac{\check{\varepsilon}}{2})$.

Now choose an integer k0 such that $\frac{1}{k_0} < \frac{\check{\varepsilon}}{2}$.

⇒ Diam(*(IFSP)*˜$(\widetilde{C_{k0}})$) $\tilde{<}$ $\frac{1}{k_0} < \frac{\check{\varepsilon}}{2}$

⇒ (*(IFSP)*˜$(\widetilde{C_{k0}})$ $\tilde{\subseteq}$ *(IFSB)*˜$(e_F x, \frac{\check{\varepsilon}}{2})$ ....(iv)

⇒ (*(IFSP)*˜$(\widetilde{C_{k0}})$ $\tilde{\subseteq}$ *(IFSP)*˜$(\widetilde{B_{n0}})$ from (ii), (iii) and (iv).

Which contradicts (i).
Hence the Lemma is proved.

Definition 3.23 Suppose (*(IFSP)*˜$(\tilde{X}_E)$, $\tilde{d}$, E) is a *(IFS)*˜metric space, then a subset *(IFSP)*˜$(\tilde{A})$ of *(IFSP)*˜$(\tilde{X}_E)$ is said to be *(IFS)*˜$\check{\varepsilon}$- net for *(IFSP)*˜$(\tilde{X}_E)$ if *(IFSP)*˜$(\tilde{A})$ is finite and *(IFSP)*˜$(\tilde{X}_E)$ $\tilde{\subseteq}$ $\cup_{e_F a \tilde{\in} IFSP(\tilde{A})}(IFSB)$˜$(e_F x, \check{\varepsilon})$.

i.e., *(IFSP)*˜$(\tilde{A})$={ $e_F a1$, $e_F a2$,....... $e_F an$}

⇒ *(IFSP)*˜$(\tilde{X}_E)$ = *(IFSB)*˜$(e_F a1, \check{\varepsilon})$ $\tilde{\cup}$ *(IFSB)*˜$(e_F a2, \check{\varepsilon})$ $\tilde{\cup}$ .... $\tilde{\cup}$ *(IFSB)*˜$(e_F an, \check{\varepsilon})$.

Definition 3.24 A *(IFS)*˜metric space (*(IFSP)*˜$(\tilde{X}_E)$, $\tilde{d}$, E) is said to be totally bounded *(IFS)*˜ metric space if it has *(IFS)*˜$\check{\varepsilon}$-net.

Theorem 3.25 If *(IFSP)*˜$(\tilde{X}_E)$ is totally bounded *(IFS)*˜ metric space *(IFS)*˜metric space,then it is bounded.

Proof. Assume that *(IFSP)*˜$(\tilde{X}_E)$ is totally bounded *(IFS)*˜ metric space.

Then it possess $\check{1}$-net.
Let *(IFSP)*˜$(\tilde{A})$={ $e_F a1$, $e_F a2$,....... $e_F an$},finite set.

⇒ *(IFSP)*˜$(\tilde{X}_E)$ = *(IFSB)*˜$(e_F a1, \check{\varepsilon})$ $\tilde{\cup}$ *(IFSB)*˜$(e_F a2, \check{\varepsilon})$ $\tilde{\cup}$ .... $\tilde{\cup}$ *(IFSB)*˜$(e_F an, \check{\varepsilon})$.

For any $e_F x$, $e_F y$ $\tilde{\in}$ *(IFSP)*˜$(\tilde{X}_E)$, $e_F x$ $\tilde{\in}$ *(IFSB)*˜$(e_F ai, \check{1})$, $e_F y$ $\tilde{\in}$ *(IFSB)*˜$(e_F aj, \check{1})$ for some i,j ∈{1,2,....n}.

$\check{d}(e_F x, e_F ai)$ $\tilde{<}$ $\check{1}$ and $\check{d}(e_F y, e_F aj)$ $\tilde{<}$ $\check{1}$.
Now $\check{d}(e_F x, e_F y)$ $\tilde{\leq}$ $\check{d}(e_F x, e_F ai)$ $\tilde{+}$ $\check{d}(e_F ai, e_F aj)$ $\tilde{+}$ $\check{d}(e_F y$,

$e_F aj)$

$\precsim \breve{2} \mp \check{d}(e_F ai, e_F aj)$

$\precsim \breve{2} \mp$ Diam($(IFSP)^{\sim}(\tilde{A})$) $\precsim \infty$ (Since $(IFSP)^{\sim}(\tilde{A})$ is finite).

This is true for all $e_F x, e_F y \,\tilde{\in}\, (IFSP)^{\sim}(\tilde{X}_E)$.

Hence $(IFSP)^{\sim}(\tilde{X}_E)$ is bounded.

Theorem 3.26 A $(IFS)^{\sim}$Sequentially compact space is totally bounded $(IFS)^{\sim}$ metric space.

Proof. Assume that $(IFSP)^{\sim}(\tilde{X}_E)$ is $(IFS)^{\sim}$Sequentially compact space.

We show that $(IFSP)^{\sim}(\tilde{X}_E)$ is totally bounded $(IFS)^{\sim}$ metric space.

Let $e_F a1 \,\tilde{\in}\, (IFSP)^{\sim}(\tilde{X}_E)$ and consider the $(IFS)^{\sim}$neighbourhood of $e_F a1$, say $(IFSB)^{\sim}(e_F a1, \breve{\varepsilon})$ for $\breve{\varepsilon} \succsim \breve{0}$.

If $(IFSP)^{\sim}(\tilde{X}_E) \,\tilde{\subseteq}\, (IFSB)^{\sim}(e_F a1, \breve{\varepsilon})$, then $(IFSP)^{\sim}(\tilde{A})= \{e_F a1\}$ forms a $(IFS)^{\sim}\breve{\varepsilon}$- net for $(IFSP)^{\sim}(\tilde{X}_E)$.

If $(IFSP)^{\sim}(\tilde{X}_E) \,\tilde{\nsubseteq}\, (IFSB)^{\sim}(e_F a1, \breve{\varepsilon})$ then we can find $e_F a2 \,\tilde{\in}\, (IFSP)^{\sim}(\tilde{X}_E)$ - $(IFSB)^{\sim}(e_F a1, \breve{\varepsilon})$.

And consider $(IFSB)^{\sim}(e_F a1, \breve{\varepsilon}) \,\tilde{\cup}\, (IFSB)^{\sim}(e_F a2, \breve{\varepsilon})$.

If $(IFSP)^{\sim}(\tilde{X}_E) \,\tilde{\subseteq}\, (IFSB)^{\sim}(e_F a1, \breve{\varepsilon}) \,\tilde{\cup}\, (IFSB)^{\sim}(e_F a2, \breve{\varepsilon})$, then $(IFSP)^{\sim}(\tilde{A})= \{e_F a1, e_F a2\}$ forms a $(IFS)^{\sim}\breve{\varepsilon}$- net for $(IFSP)^{\sim}(\tilde{X}_E)$.

If $(IFSP)^{\sim}(\tilde{X}_E) \,\tilde{\nsubseteq}\, (IFSB)^{\sim}(e_F a1, \breve{\varepsilon}) \,\tilde{\cup}\, (IFSB)^{\sim}(e_F a2, \breve{\varepsilon})$,

then we can find $e_F a3 \,\tilde{\in}\, (IFSP)^{\sim}(\tilde{X}_E)$ – {$(IFSB)^{\sim}(e_F a1, \breve{\varepsilon}) \,\tilde{\cup}\, (IFSB)^{\sim}(e_F a2, \breve{\varepsilon})$}.

If the process continued indefinitely we get a sequence $\{e_F a_{n,k}\}$ in $(IFSP)^{\sim}(\tilde{X}_E)$ such that $\check{d}(e_F ai, e_F aj) \succsim \breve{\varepsilon}$.

Whcih shows that the sequence $\{e_F a_{n,k}\}$ has no convergent subsequence.
Which is a contradiction to the hypothesis.

So this process must terminate after a finite stage.
& we must have $(IFSP)^{\sim}(\tilde{X}_E) \,\tilde{\subseteq}\, (IFSB)^{\sim}(e_F a1, \breve{\varepsilon}) \,\tilde{\cup}\, (IFSB)^{\sim}(e_F a2, \breve{\varepsilon})$........... $\tilde{\cup}\, (IFSB)^{\sim}(e_F an, \breve{\varepsilon})$ for some n.

Hence $(IFSP)^{\sim}(\tilde{A})$ ={ $e_F a1$, $e_F a2$,....... $e_F an$} forms a $(IFS)^{\sim}\breve{\varepsilon}$- net.

Hence $(IFSP)^{\sim}(\tilde{X}_E)$ is totally bounded $(IFS)^{\sim}$ metric space.

Theorem 3.27 A $(IFS)^{\sim}$sequential compact space is $(IFS)^{\sim}$compact.

Proof. Assume that $(IFSP)^{\sim}(\tilde{X}_E)$ is $(IFS)^{\sim}$Sequentially compact space.

We show that $(IFSP)^{\sim}(\tilde{X}_E)$ is $(IFS)^{\sim}$compact space.

Let $\{(IFSP)^{\sim}(A_\alpha)\}_{\alpha\in\Delta}$ be a $(IFS)^{\sim}$open cover for $(IFSP)^{\sim}(\tilde{X}_E)$.

Then by Lemma 3.22 this $(IFS)^{\sim}$open cover has a *LIFSN* number, Say $\breve{\alpha}$.

∴ For any $(IFSP)^{\sim}(\tilde{A}) \,\tilde{\subseteq}\, (IFSP)^{\sim}(\tilde{X}_E)$ with Diam($(IFSP)^{\sim}(\tilde{A})$) $\precsim \breve{\alpha}$.

$(IFSP)^{\sim}(\tilde{A})$ lies in exactly one member of $\{(IFSP)^{\sim}(A_\alpha)\}_{\alpha\in\Delta}$.......(i)

Let $\breve{\varepsilon} = \frac{\breve{\alpha}}{2} \succsim \breve{0}$.
Since is $(IFS)^{\sim}$sequential compact it is $(IFS)^{\sim}$totally bounded space.

∴ it has a $(IFS)^{\sim}\breve{\varepsilon}$- net.

$(IFSP)^{\sim}(\tilde{A})$ ={ $e_F a1$, $e_F a2$,....... $e_F an$} be a $(IFS)^{\sim}\breve{\varepsilon}$- net.

∴ $(IFSP)^{\sim}(\tilde{X}_E) \,\tilde{\subseteq}\, (IFSB)^{\sim}(e_F a1, \breve{\varepsilon}) \,\tilde{\cup}\, (IFSB)^{\sim}(e_F a2, \breve{\varepsilon})$........... $\tilde{\cup}$ $(IFSB)^{\sim}(e_F an, \breve{\varepsilon})$........(ii)

⇒ Diam($(IFSB)^{\sim}(e_F ai, \breve{\varepsilon})$) $\precsim 2\breve{\varepsilon} \precsim \breve{\alpha}$ for i = 1, 2, ... n.

⇒ By (i) there is exactly one $\alpha, i \in \Delta$ such that $(IFSB)^{\sim}(e_F ai, \breve{\varepsilon}) \,\tilde{\subseteq}\, IFSP(A_{\alpha,i})$ for i = 1, 2, ... n......(iii)

From (ii) and (iii) $(IFSP)^{\sim}(\tilde{X}_E) \,\tilde{\subseteq}\, IFSP(A_{\alpha,1}) \,\tilde{\cup}\, IFSP(A_{\alpha,2})$ $\tilde{\cup}$..... $\tilde{\cup}\, IFSP(A_{\alpha,n})$.

⇒ $\{(IFSP)^{\sim}(A_{\alpha,1}), (IFSP)^{\sim}(A_{\alpha,2}), ..., (IFSP)^{\sim}(A_{\alpha,n})\}$ forms a $(IFS)^{\sim}$open cover for $(IFSP)^{\sim}(\tilde{X}_E)$.

Hence $(IFSP)^{\sim}(\tilde{X}_E)$ is $(IFS)^{\sim}$compact space.

Definition 3.28 Let ($(IFSP)^{\sim}(\tilde{X}_E)$, $\widetilde{d1}$, E) and ($(IFSP)^{\sim}(\tilde{Y})$, $\widetilde{d2}$, E) be two $(IFS)^{\sim}$metric spaces. A mapping $\tilde{f}$: ($(IFSP)^{\sim}(\tilde{X}_E)$, $\widetilde{d1}$, E) → ($(IFSP)^{\sim}(\tilde{Y})$, $\widetilde{d2}$, E) is said to be $(IFS)^{\sim}$uniformly continuous on $(IFSP)^{\sim}(\tilde{X})$ if for each $\breve{\varepsilon} \succsim \breve{0}$ ∃ $\breve{\delta} \succsim \breve{0}$ such that $\widetilde{d1}(e_F x, e_F y) \precsim \breve{\delta} \Rightarrow \widetilde{d2}(\tilde{f}(e_F x), \tilde{f}(e_F y)) \precsim \breve{\varepsilon}$.This is true forevery $e_F x, e_F y \,\tilde{\in}\, (IFSP)^{\sim}(\tilde{X})$.

Theorem 3.29 Suppose ($(IFSP)^{\sim}(\tilde{X}_E)$, $\tilde{d}$, E) is a $(IFS)^{\sim}$compact space and $\tilde{f}$: ($(IFSP)^{\sim}(\tilde{X}_E)$, $\widetilde{d1}$, E) ⟶ ($(IFSP)^{\sim}(\tilde{Y})$, $\widetilde{d2}$, E) is $(IFS)^{\sim}$continuous where ($(IFSP)^{\sim}(\tilde{Y})$, $\widetilde{d2}$, E) is any arbitrary $(IFS)^{\sim}$space then $\tilde{f}$ $(IFS)^{\sim}$uniformly continuous.

Proof. Given ($(IFSP)^{\sim}(\tilde{X}_E)$, $\tilde{d}$, E) is a $(IFS)^{\sim}$compact space and

$\tilde{f}$: ($(IFSP)^{\sim}(\tilde{X}_E)$, $\widetilde{d1}$, E) ⟶ ($(IFSP)^{\sim}(\tilde{Y})$, $\widetilde{d2}$, E) is $(IFS)^{\sim}$continuous.

Now we prove that $\tilde{f}$ is $(IFS)^{\sim}$uniformly continuous.

For given $\breve{\varepsilon} \succsim \breve{0}$ and for any $e_F x \,\tilde{\in}\, (IFSP)^{\sim}(\tilde{X}_E)$.

Let *(IFSP)*$\tilde{}$$(\widetilde{Vx})$ = *(IFSB)*$\tilde{}$$(\tilde{f}(e_F x), \frac{\breve{\varepsilon}}{2})$

Since $\tilde{f}$ is *(IFS)*$\tilde{}$continuous the inverse image $\tilde{f}^{-1}$(*(IFSP)*$\tilde{}$$(\widetilde{Vx})$) of *(IFSP)*$\tilde{}$$(\widetilde{Vx})$ is in *(IFSP)*$\tilde{}$$(\tilde{X}_E)$.

Let *(IFSP)*$\tilde{}$$(\widetilde{Gx})$ = $\tilde{f}^{-1}$(*(IFSP)*$\tilde{}$$(\widetilde{Vx})$).

⇛ For any $e_F x$ $\tilde{\in}$ *(IFSP)*$\tilde{}$$(\tilde{X})$ we have a *(IFS)*$\tilde{}$open set *(IFSP)*$\tilde{}$$(\widetilde{Gx})$ in *(IFSP)*$\tilde{}$$(\tilde{X}_E)$.

So $\{IFSP(\widetilde{Gx})\}_{e_F x \,\tilde{\in}\, IFSP(\tilde{X})}$ forms a *(IFS)*$\tilde{}$open cover for *(IFSP)*$\tilde{}$$(\tilde{X}_E)$.

And *(IFSP)*$\tilde{}$$(\tilde{X}_E)$ is *(IFS)*$\tilde{}$compact ⇛ *(IFSP)*$\tilde{}$$(\tilde{X}_E)$ is *(IFS)*$\tilde{}$sequential compact space by Theorems 3.17 and 3.18.

⇛ The *(IFS)*$\tilde{}$open cover has a *IFS*L number by Lemma 3.22,Let it be $\breve{\delta}$.

∴ For any *(IFSP)*$\tilde{}$$(\tilde{A})$$\tilde{\subseteq}$ *(IFSP)*$\tilde{}$$(\tilde{X}_E)$ with Diam(*(IFSP)*$\tilde{}$$(\tilde{A})$) $\tilde{\lesssim}$ $\breve{\delta}$.

(IFSP)$\tilde{}$$(\tilde{A})$ lies exactly in one *(IFSP)*$\tilde{}$$(\widetilde{Gx})$ such that $e_F x$ $\tilde{\in}$ *(IFSP)*$\tilde{}$$(\tilde{X}_E)$.

Let $e_F x$, $e_F y$ $\tilde{\in}$ *(IFSP)*$\tilde{}$$(\tilde{X}_E)$ with $\widetilde{d1}(e_F x, e_F y)$ $\tilde{<}$ $\breve{\delta}$.

Then the two elements set *(IFSP)*$\tilde{}$$(\tilde{A})$ = { $e_F x$, $e_F y$} with Diam(*(IFSP)*$\tilde{}$$(\tilde{A})$) $\tilde{\lesssim}$ $\breve{\delta}$.

⇛ *(IFSP)*$\tilde{}$$(\tilde{A})$ lies exactly in one *(IFSP)*$\tilde{}$$(\widetilde{Gx0})$.

⇛ *(IFSP)*$\tilde{}$$(\tilde{A})$ $\tilde{\subseteq}$ *(IFSP)*$\tilde{}$$(\widetilde{Gx0})$.

⇛ $e_F x$, $e_F y$ $\tilde{\in}$ *(IFSP)*$\tilde{}$$(\widetilde{Gx0})$.

⇛ $e_F x$, $e_F y$ $\tilde{\in}$ $\tilde{f}^{-1}$(*(IFSP)*$\tilde{}$$(\widetilde{Vx0})$).

⇛ $\tilde{f}(e_F x), \tilde{f}(e_F y)$ $\tilde{\in}$ *(IFSP)*$\tilde{}$$(\widetilde{Vx0})$.

⇛ $\tilde{f}(e_F x), \tilde{f}(e_F y)$ $\tilde{\in}$ *(IFSB)*$\tilde{}$$(\tilde{f}(e_F x0), \frac{\breve{\varepsilon}}{2})$.

⇛ $\widetilde{d2}(\tilde{f}(e_F x), \tilde{f}(e_F x0)) \tilde{<} \frac{\breve{\varepsilon}}{2}$ and $\widetilde{d2}(\tilde{f}(e_F y), \tilde{f}(e_F x0)) \tilde{<} \frac{\breve{\varepsilon}}{2}$.
Now $\widetilde{d2}(\tilde{f}(e_F x), \tilde{f}(e_F y))$ $\tilde{\leq}$ $\widetilde{d2}(\tilde{f}(e_F x), \tilde{f}(e_F x0))$ $\tilde{+}$ $\widetilde{d2}(\tilde{f}(e_F x0), \tilde{f}(e_F y))$ $\tilde{<} \frac{\breve{\varepsilon}}{2} \tilde{+} \frac{\breve{\varepsilon}}{2} = \breve{\varepsilon}$
Hence the theorem.

Conclusion

In this paper we have defined *(IFS)*$\tilde{}$sequential compact space, *(IFS)*$\tilde{}$soft real number, *(IFS)*$\tilde{}$point.This study contributes some important theorems and properties on the concepts of Lebesgue *(IFS)*$\tilde{}$ number, *(IFS)*$\tilde{}$$\breve{\varepsilon}$- net ,totally bounded *(IFS)*$\tilde{}$metric space.At the end the concept of *(IFS)*$\tilde{}$uniformly continuous mapping is defined and an important theorem is proved.

References

1. Molodtsov, Soft set theory first results, Comput. Math. Appl., 37(4-5)(1999), 19-31.
2. Zadeh, Lotfi A. "Fuzzy sets." *Information and control* 8.3 (1965): 338-353.
3. Tanay, Bekir, and M. Burc Kandemir. "Topological structure of fuzzy soft sets."*Computers & Mathematics with Applications* 61.10 (2011): 2952-2957.
4. P.K. Maji, R. Biswas, A.R. Roy, Fuzzy soft sets, J. Fuzzy Math. 9 (2001) 589–602.
5. Jiang, Yuncheng, Yong Tang, and Qimai Chen. "An adjustable approach to intuitionistic fuzzy soft sets based decision making." *Applied Mathematical Modelling* 35.2 (2011): 824-836.
6. Gunduz, Cigdem, and Sadi Bayramov. "Intuitionistic fuzzy soft modules."*Computers & Mathematics with Applications* 62.6 (2011): 2480-2486.
7. Bayramov, Sadi, and Cigdem Gunduz. "On Intuitionistic Fuzzy Soft Topological Spaces." *II International Conference*.
8. Mahanta, J., and P. K. Das. "Results on fuzzy soft topological spaces." *arXiv preprint arXiv:1203.0634* (2012).
9. Ejegwa, P. A., A. J. Akubo, and O. M. Joshua. "Intuitionistic fuzzy set and its application in career determination via normalized Euclidean distance method."*European Scientific Journal* 10.15 (2014).