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An Algorithm for 2-Tuple Total Domination Number in Circulant Graphs

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Abstract
This paper studies perfect 2-tuple total domination number for the circulant graphs $Cir(n, A)$, *where* $A = \{1,2,\ldots,x, n-1, n-2,\ldots,n-x\}$ *and* $x \le \left\lfloor \frac{n-1}{2} \right\rfloor$ *from an algorithmic point of view.*

Keywords: *Circulant graphs, domination ,2-tuple total domination number.*

Introduction

In this paper, we follow the notation of [3]. Domination is an important property in the design of efficient computer interconnection networks. We studied the perfect 2-dominating sets for circulant graphs $Cir(n, A)$,where

$A = \{1,2,\ldots,x, n-1, n-2,\ldots,n-x\}$ and $x \le \left\lfloor \frac{n-1}{2} \right\rfloor$ [9].

A vertex subset of a graph $G = (V, E)$, is called a dominating set if every vertex v not in S, is adjacent to a vertex in S. The domination number of G, denoted by $\gamma(G)$, is the minimum cardinality of a dominating set in G and a corresponding dominating set is called a $\gamma - set$[3]. A vertex subset S is said to be an efficient dominating set if for every vertex v, $|N[v] \cap S| = 1$ [1]. Let $k(\ge 1)$ be an integer. A vertex subset S is said to be $k-$dominating($k-$tuple total dominating) set if for each vertex $v \in V - S(v \in V), |N(v) \cap S| \ge k$. The $k-$domination number($k-$tuple total domination number) of G is the minimum cardinality of a $k-$dominating ($k-$tuple total dominating) set denoted by $\gamma_k(G)(\gamma_{\times k,t}(G))$ [4].

A $k-$dominating set S is said to be independent $k-$dominating set if no two vertices in S are adjacent. A $k-$dominating set($k-$tuple total dominating set) S is said to be perfect if for every vertex $v \in V - S(v \in V), |N(v) \cap S| = k$. A perfect and independent $k-$dominating set is called as efficient $k-$dominating set.

Cayley graphs have been an important class of graphs in the study of interconnection networks for parallel and distributed computing. Let $(\Gamma, *)$ be a finite group and e be its identity. Let A be a generating set of Γ such that $e \notin A$ and $a^{-1} \in A$ for all $a \in A$. Then the Cayley graph is defined by $G = (V, E)$, where $V = \Gamma$ and $E = \{(x, x*a)/x \in V, a \in A\}$, denoted by $Cay(\Gamma, A)$. Circulant graphs are special case of Cayley graphs when $\Gamma = (Z_n, \oplus_n)$, where $\oplus_n$ is the operation addition modulo n [10].

The purpose of the paper to study an algorithm for perfect 2-tuple total domination number for these circulant graphs.

2-tuple total domination number

The author studied the perfect 2-dominating sets for thecirculant graphs $Cir(n, A)$, where

$A = \{1,2,\ldots,x,n-1,n-2,\ldots,n-x\}$ and $x \le \left\lfloor \frac{n-1}{2} \right\rfloor$.

In this collection of graphs, the perfect 2-tuple total domination number $\gamma_{\times 2t}$ has been obtained [9].

This section gives some of the results on perfect 2-tuple total domination number of $Cir(n, A)$ [9].

Lemma: 2.1 *Let $n(\ge 3)$, x be integers. Let $G = Cir(n, A)$ be a circulant graph with $A = \{1,2,\ldots,x,n-1,n-2,\ldots,n-x\}$ and $x \le \left\lfloor \frac{n-1}{2} \right\rfloor$. If x divides n, then G has a perfect 2-tuple total dominating set.*

Lemma: 2.2 *Let $n(\ge 3)$, x be integers. Let $G = Cir(n, A)$ be a circulant graph with $A = \{1,2,\ldots,x,n-1,n-2,\ldots,n-x\}$ and $4 \le x \le \left\lfloor \frac{n-1}{2} \right\rfloor$. If G has a perfect 2-tuple total dominating set, then x divides n.*

Theorem: 2.3 *Let $G = Cir(n, A)$ be a circulant graph with $A = \{1,2,\ldots,x,n-1,n-2,\ldots,n-x\}$ and $4 \le x \le \left\lfloor \frac{n-1}{2} \right\rfloor$. Then G has a perfect 2-tuple total dominating set if and only if x divides n.*

Lemma: 2.4 *Let $H(\ne e)$ be a subgroup of Z_n. Then H is a perfect 2-tuple total dominating set for the circulant graph $Cir(n, A)$ for some suitable generating set A of Z_n.*

Lemma: 2.5 *Let G = Cir(n, A) be a circulant graph with $A = \{1,2,\ldots,x,n-1,n-2,\ldots,n-x\}$ and $x \le \left\lfloor \frac{n-1}{2} \right\rfloor$. Then $\gamma_{\times 2t}(G) = \left\lceil \frac{n}{x} \right\rceil$.*

Proof: Suppose G has a 2-tuple total dominating set D.

Let $n = gx + j$ for some integers $g(\ge 1)$ and j with $0 \le j \le x-1$.

Without loss of generality, assume that $0 \in D$. As discussed in Lemma 3.2 [9], we have $0, x, 2x \ldots gx \in D$, where $g+1 = \left\lceil \frac{n}{x} \right\rceil$. Hence $\gamma_{\times 2t}(G) \ge \left\lceil \frac{n}{x} \right\rceil$.

Let $S = \{0, x, 2x \ldots gx\}$ and $v \in V(G)$.

Case 1: $ix < v < (i+1)x$ for some integer i with $0 \le i \le g-1$.

In this case, v is dominated by ix and $(i+1)x$.

Case 2: $gx < v \le n-1$.

In this case, v is dominated by both 0 and gx.

Case 3: $v \in D$ and $v = ax$ for some $1 \le a \le g-1$.

In this case, v is dominated by both $(a-1)x, (a+1)x \in D$.

Case 4: $v \in D$ and $v = gx$ or $v = 0$.

If $v = gx$, then it is dominated by both $(g-1)x, 0 \in D$.

If $v = 0$, then it is dominated by both $gx, x \in D$.

Hence $\gamma_{\times 2t}(G) \le |S| = \left\lceil \frac{n}{x} \right\rceil$ and hence $\gamma_{\times 2t}(G) = \left\lceil \frac{n}{x} \right\rceil$.

Algorithm for 2-tuple total domination number

The main result of this section is an algorithm for the $2-$tuple domination number in $cir(n, A)$ based on the lemma 2.5.

Algorithm

Input: A circulant graph G=cir(n,A), integer g,x,j, A={1,2,...x,n-1,n-2,...n-x} and $x \le \left\lfloor \frac{n-1}{2} \right\rfloor$

Output: A 2-tuple total dominating set H of G

Begin

initialize n=0

initialize

n=gx+j $/*Let (g \ge 1) and (j \ge 0) and (j \le x-1)*/$

if

(0∈D) $/*since\ 0 \in D, 0, x, 2x, \ldots gx\ belongs\ to\ D*/$

H >= $\left\lceil \frac{n}{x} \right\rceil$ $/*H = \gamma_{xt}(G)*/$

else

for v in V(G) do

Switch
case (v > ix) and (v < (i+1)x)
v is dominated by ix and (i+1)x
case(v > gx) and (v <= n-1)
v is dominated by 0 and gx
case (v ∈ D) and (v = ax)
/*1≤a≤g-1*/
v is dominated by both (a-1)x,(a+1)x
case (v ∈ D) and (v = gx)
v is dominated by both x and (g-1)x
case (v ∈ D) and (v = 0)
v is dominated by both x,gx belongs to D
/*x belongs to D*/
end switch

H <= $\left\lceil \frac{n}{x} \right\rceil$

end

if ($0 \in D$ and $v \in V(G)$)

H= $\left\lceil \frac{n}{x} \right\rceil$

end

end

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