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Effect of Monopole field on the Gravitational Collapse of Husain Space-Time

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Abstract
We study the effect of the monopole field on the occurrence of the naked singularities arising in Husain space-time. For an appropriate choice of the arbitrary functions, the outgoing radial null geodesics, emanating from the central singularity has one or more positive roots. The occurrence of naked singularity violates the cosmic censorship hypothesis.

Key Words: Cosmic censorship hypothesis, Naked singularity, Gravitational collapse

Introduction

The cosmic censorship hypothesis (CCH)[1] implies that the singularity formed in the generic gravitational collapse of a star is always a black hole. A singularity is naked if it is visible to an external observer. There has been extensive research work on naked singularities and black holes, and on the validity of CCH [2-4]. A. Wang [5] introduced a more general family of space-times which covers monopole solutions and Husain solution. It has been explained in Ref. [6] that monopoles are formed due to a guage-symmetry breaking and have many properties of elementary particles. Most of their energy is concentrated in a small region near the core. We have studied the gravitational collapse of Husain solution and found that this solution admits naked singularities under certain conditions on the mass function[7]. In the present work the aim is to study the occurrence of the naked singularities in Husain space-time after the introduction of the external field- the monopole field.

The paper is organized as follows: In Sec. II, we describe the monopole Husain space-time. In Sec. III, we discuss the nature of the singularities arising in this space-time. We conclude the paper in Sec. IV.

Introduction of monopole field in the Husain solution

The general spherically symmetric line element [5] is

$$ds^2 = -\left(1-\frac{2m(u,r)}{r}\right)du^2 + 2\,du dr + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) \qquad (1)$$

where, $m(u,r)$ is the mass function which is related to gravitational energy within a given radius r.

Denoting $\{x^1, x^2, x^3, x^4\} \equiv \{u, r, \theta, \varphi\}$,

$\dot{}\ \equiv$ partial derivatives with respect to u and

$'\equiv$ partial derivatives with respect to r;

the non-zero components of the Einstein tensor for the above compute to

$$G_0^0 = G_1^1 = -\frac{2m'}{r^2}, \quad G_0^1 = 2\frac{\dot{m}}{r^3},$$

$$G_2^2 = G_3^3 = -\frac{m''}{r} \qquad (2)$$

The Einstein field equations are

$$G_{\mu\nu} = \kappa T_{\mu\nu} \qquad (3)$$

where,

$$T_{\mu\nu} = \sigma l_\mu l_\nu + (\rho + P)(l_\mu n_\nu + l_\nu n_\mu) + P g_{\mu\nu} \qquad (4)$$

$$l_\mu = \delta_\mu^0, \quad n_\mu = \frac{1}{2}\left(1 - \frac{2m(u,r)}{r}\right)\delta_\mu^0 - \delta_\mu^1,$$

$$l_\lambda l^\lambda = n_\lambda n^\lambda = 0, \; l_\lambda n^\lambda = -1 \qquad (5)$$

l_μ and n_μ being the null vectors.

The energy-momentum tensor for this fluid belongs to type II fluids. The energy conditions for such type of fluids are given by [5]

(i) The weak and strong energy conditions:

$$\sigma > 0, \ \rho \geq 0, \ P \geq 0 \tag{6}$$

(ii) The dominant energy conditions:

$$\sigma > 0, \rho \geq P \geq 0 \tag{7}$$

Using Eqs. (2)–(4) we obtain the following expressions for σ, ρ and P

$$\sigma = 2\frac{\dot{m}(u,r)}{\kappa r^2}, \ \rho = \frac{2m'(u,r)}{\kappa r^2}, \ P = -\frac{m''(u,r)}{\kappa r} \tag{8}$$

A. Wang[5] has defined the monopole solution in the form

$$2m_1(u,r) = hr^2 \tag{9}$$

where h is an arbitrary positive constant satisfying $0 < h < 1$.

Husain[8] has imposed the condition $P = k\rho$, on the null fluid and defined the mass function

$$m_2(u,r) = f(u) - \frac{g(u)}{(2k-1)r^{2k-1}}, k \neq 1/2$$
$$= f(u) + g(u)\ln r, \quad k = 1/2 \tag{10}$$

where, $f(u)$ and $g(u)$ are arbitrary functions. When $k = 1/2$, it can be seen that the energy conditions are always violated for sufficiently small r and hence we shall consider the case $k \neq 1/2$. [8]

A linear superposition of particular solutions is also a solution of the Einstein field equations. In particular, combining the mass functions (9) and (10) one can write

$$m(u,r) = f(u) - \frac{g(u)}{(2k-1)r^{2k-1}} + \frac{hr}{2}, k \neq 1/2$$
$$= f(u) + g(u)\ln r + \frac{hr}{2}, \quad k = 1/2 \tag{11}$$

With the choice of the mass function (11), the quantities σ, ρ and Pcan be expressed as

$$\sigma = \frac{2}{\kappa r^2}\left[\dot{f}(u) - \frac{\dot{g}(u)}{(2k-1)r^{2k-1}}\right],$$

$$\rho = \frac{2}{\kappa r^2}\left[\frac{(2k-1)g(u)}{r^{2k}} + \frac{h}{2}\right],$$
$$P = \frac{2k(2k-1)g(u)}{kr^{2k+2}} \tag{12}$$

Nature of the singularities arising in monopole-Husain space-time

We use the technique developed in Ref. [2] to analyze the nature of singularity. The existence of a naked singularity can be determined by examining the behavior of radial null geodesics. If they terminate at the singularity with a positive tangent vector, then the singularity is naked. If the tangent vector is not positive in the limit as one approaches the singularity, then the singularity is not naked or covered. Inserting the mass function (11) into Eq. (1), we write the monopole-Husain space-time as

$$ds^2 = -\left(1 - h - \frac{2f(u)}{r} - \frac{2g(u)}{(2k-1)r^{2k}}\right)du^2 + 2\,dudr + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) \tag{13}$$

Above equation has a solution if $f(u) \propto u$ and $g(u) \propto u^{2k}$. In particular, let us take $f(u) = \alpha u$ and $(u) = \beta u^{2k}$, where α and β are some positive constants. As k takes values between 0 and 1, let us choose $k = 3/4$. Then the mass function (11) becomes

$$m(u,r) = \alpha u - \frac{2\beta u^{3/2}}{r^{1/2}} + \frac{hr}{2} \tag{14}$$

Hence the space-time (13) becomes

$$ds^2 = -\left(1 - h - \frac{2\alpha u}{r} - \frac{4\beta\, u^{\frac{3}{2}}}{r^{\frac{3}{2}}}\right)du^2 + 2\,dudr + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) \tag{15}$$

Along radial null geodesics, we have

$$\xi^a K_a = uK_u + rK_r \tag{16}$$

where,

$$\xi^a = u\frac{\partial}{\partial u} + r\frac{\partial}{\partial r}, \ K^u = \frac{\partial u}{\partial k}, \ K^r = \frac{\partial r}{\partial k} \tag{17}$$

Defining the function $R(u,r) = rK^u, \ \ Y = u/r$, we obtain the deferential equation

$$\frac{dR}{dk} = \frac{R^2}{2r^2}\left(1 - \frac{4m}{r}\right) \tag{18}$$

Using Eq. (16), we obtain its solution

$$R = \frac{2C}{2 - Y + hY + 2\alpha Y^2 - 4\beta\, Y^{5/2}} \tag{19}$$

The equation of the radial null geodesics for the metric (15) is given by

$$\frac{du}{dr} = \frac{2}{\left(1-h-\frac{2\alpha u}{r}-\frac{4\beta\, u^{3/2}}{r^{3/2}}\right)} \qquad (20)$$

which has a singularity at $r = 0,\ u = 0.$

The geodesic tangent is uniquely defined and exist at this point[9], if

$$Y_0 = \lim_{u\to 0\, r\to 0} \frac{u}{r} = \lim_{u\to 0\, r\to 0} \frac{du}{dr}$$
$$= \frac{2}{\left(1-h-2\alpha\, Y_0 + 4\, \beta\, Y_0^{3/2}\right)} \qquad (21)$$

Setting $Y_0 = z^2$, above equation gives

$$4\beta z^5 - 2\alpha z^4 + (1-h)z^2 - 2 = 0 \qquad (22)$$

This is an equation of odd degree with the coefficient of the first term positive and its last term negative, hence the above equation has at least one positive root. In particular, for $h = 0.5,\ \alpha = 0.01$ *and* $\beta = 0.001,$ we obtain $z = 2.119.$ Back substitution gives $Y_0 = 4.459$ as the root of Eq. (19). The existence of positive root to Eq. (19) suggests that the gravitational collapse ends into a naked singularity.

Conclusion

The introduction of monopole field does not change the nature of the singularities arising in the gravitational collapse of Husain space-time. Thus the CCH violation in the gravitational collapse of Husain space-time continues when we make transition to the monopole Husain space-time.

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