IJETST- Volume||01||Issue||08||Pages 1264-1266||October||ISSN 2348-9480 2014

Open access Journal International Journal of Emerging Trends in Science and Technology

Fractional Order Control System

Authors

Swati Singh, Ankita Kosti

Department of Electrical Engineering, Sri Ram Institute of Technology, Jabalpur

Emil: *singh.swati2703@gmail.com*

ABSTRACT

This paper includes the study of fractional order PID control system. In this firstly the study of step input signal in time domain for getting transient response using normal control system is done. Then with the use of fractional order system same analysis is done and the two results are being compared. With the use of fractional order control system, improved response is obtained.

INTRODUCTION

In the classical integer models description of memories and hereditary properties of various materials and processes were not taken into the account. While in fractional order controllers these parameters were fulfilled and thus it became its major advantage over classical integer controllers. Fractional Order derivatives became more advantageous in the modeling of electrical, electro-mechanical and mechanical properties of real materials and in other fields. Based on description of properties in terms of fractional derivatives, the mathematical modeling and simulation of systems and processes, helps to solve the differential equations of fractional order to obtain the response of particular output for a given input. With the help of fractional order methods it is easy to describe and model a real object more accurately than classical integer methods. In this paper different definitions for fractional order controller are included. The simulation has been done for time domain and frequency domain and results for transfer function are obtained.

FRACTIONAL ORDER CONTROL SYSTEM

Consider a simple feedback control system where G(s) is the transfer function of the controlled system, $G_c(s)$ is the transfer function of the controller, E(s) is an error,

W(s) is an input, U(s) is controller's output and Y(s) is system's output.

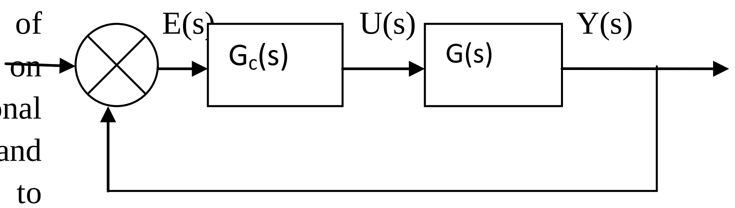


Figure 1.1

Let us assume the transfer function for the above figure be:

$$G_n(s)=\frac{1}{a_n s^{\beta_n}+a_{n-1}s^{\beta_{n-1}}+\dots+a_1 s^{\beta_1} a_0 s^{\beta_0}}$$

where

β_k, (k=0,1,2,3,…,n) is an arbitrary real number,

$\beta_n > \beta_{n-1} > \dots > \beta_1 > \beta_0$

a_k, (k=0,1,2,3,…,n) is an arbitrary constant.

MATHEMATICAL OVERVIEW

Fractional order differentiations and integrations have different definitions and some of them are directly extended from integer-order calculus.

Definition 1:

This definition is a general extension of the integer order Cauchy formula $D^{\gamma} f(t) = \frac{\Gamma(\gamma+1)}{2\pi j} \int \frac{f(\tau)}{(\tau-t)^{\gamma+1}} d\tau$ (2.1)

Definition 2:

$${}_aD_t^{\alpha} = \lim_{n\to\infty} \left(\frac{1}{h}\right)^{\alpha} \sum_{j=0} \left[\frac{t-a}{h}\right] (-1)^j \binom{\alpha}{j} f(t-jh) \quad (2.2)$$

This definition is known as *Grunwald–Letnikov definition*. Here $w\binom{\alpha}{j}$ represents the coefficient of the polynomial $(1-z)^{\alpha}$

Definition 3:

It gives an integration equation for fractional order. This equation is known as *Riemann–Liouville Fractional Order differentiation*.

$${}_aD_t^{-\alpha} f(t) = \left(\frac{1}{r(\alpha)}\right) \int_a^t (t-\tau)^{\alpha-1} f(\tau) d\tau \quad (2.3)$$

where $0 < \alpha < 1$ and a is the initial time instance, often assumed to be zero, i.e., $a = 0$. The subscripts a and t on both sides of D represent, respectively, the lower and upper bounds in the integration

It is the most widely used equation in fractional order calculus.

Definition 4:

$${}_0D_t^{\gamma} y(t) = \left(\frac{1}{r(\alpha)}\right) \int_0^t \left(\frac{y(\tau)}{(t-\tau)^{\gamma+1}}\right) d\tau, \gamma < 1 \quad (2.4)$$

This equation is known as *Caputo's definition of Fractional Order integration.*

Equation 2.4 is derived from Caputo's Equation which is defined below.

$${}_aD_t^{\alpha} f(t) = \frac{1}{r(1-\gamma)} \int_0^t \frac{y^{(m+1)}}{(t-\tau)^{\gamma}} (\tau) d(\tau) \quad (2.5)$$

RESULT

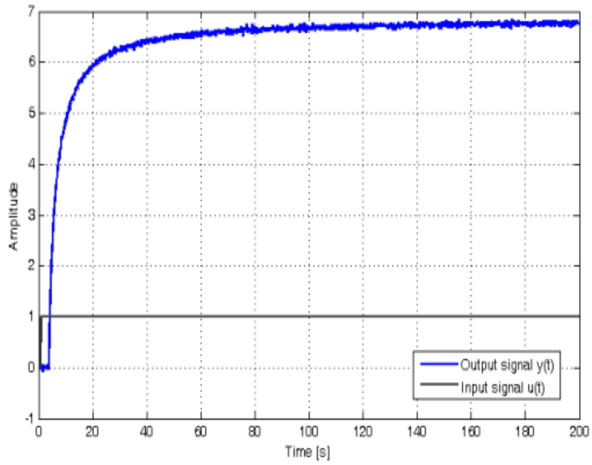
Control Design:

Considering a system having fractional order dynamics, as well as significantly input-output delay and is described by a transfer function:

$$G(s) = \frac{6.902}{1+2.686s^{0.73}} e^{-2.9s}$$

Let the actuator of this system saturates at $u = \pm 1$.

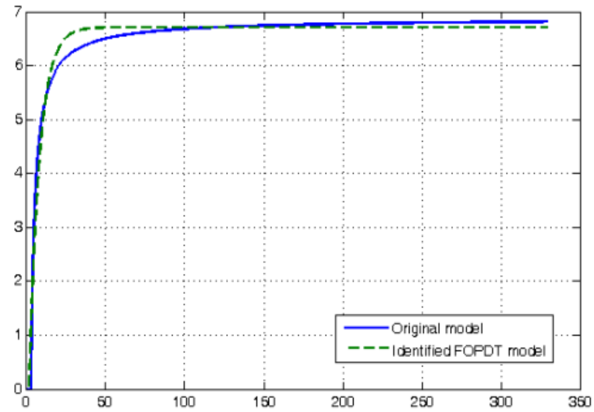
Time domain data for step input



Thus fractional order model can be identified as:

$$G(s) = \frac{6.70913}{6.53847s+1} e^{-1.57628s}$$

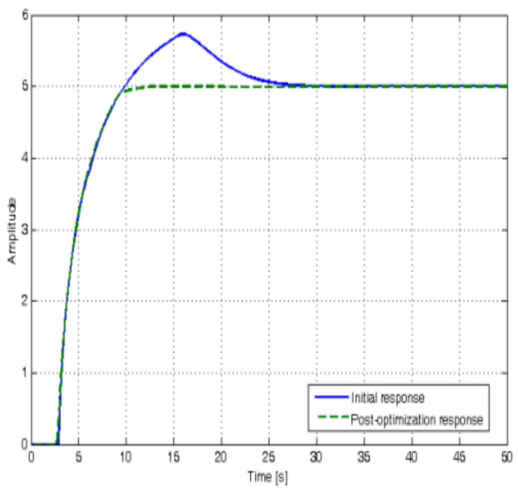
Thus the result for approximation for above transfer function is:



Fractional-order PID controller optimization:

Simulink was used to evaluate the transient response of any system.

$$C(s) = 2.0717 + \frac{0.10262}{s^{0.94962}} + 0.47143 s^{0.89562}$$



Above figure shows the result for optimization. This figure clearly shows the improvement of the control system.

CONCLUSION

The transient response of classical integer controllers has weak response than fractional order controllers. That is fractional order controllers are more optimized and give better response.

REFERENCES

1. N. Abe and K. Yamanaka, "Smith predictor control and internal model control – a tutorial," in *SICE 2003 Annual Conference*, vol. 2, 2003, pp. 1383–1387.
2. D. Xue, Y.Q. Chen MATLAB Solutions to Advanced Applied Mathematical Problems. Beijing: Tsinghua University Press, 2004.
3. R. Hilfer "Applications of Fractional Calculus in Physics", Singapore: World Scientific,2000
4. Podlubny Fractional Differential Equations. San Diego: Academic Press, 1999.
5. Chunna Zhao, Xiangde Zhang "Application of fractional order PID controller to position servomechanism" China, 2008.
6. K.B. Oldham and J. Spanier, *The Fractional Calculus*, Academic Press, New York and London, 1974
7. Podlubny, "Fractional-order systems and fractional-order controllers," *The Academy of Sciences Institute of Experimental Physics, UEF-03-94* Kosice, Slovak Republic, 1994.
8. Oustaloup, et al. "Frequency-band complex noninteger differentiator: characterization and synthesis," *IEEE Trans. Circuits Syst. I*, vol. 47, pp. 25-39, 2000.
9. C.N. Zhao, D.Y. Xue and Y.Q. Chen, "A fractional order PID tuning algorithm for a class of fractional order plants," *Proc of the IEEE International Conference on Mechatronics and Automation,* Niagara Falls, Canada , 2005, pp. 216-221.