Non-Local Image De-noising and Post Processing Using KL Transform

Authors

Raghu J
M.Tech (SP), Dept. of ECE
Reva ITM, Bangalore
India – 560 064
raghujmandya@gmail.com

Dr. Bharathi S H
Professor, Dept. of ECE
Reva ITM, Bangalore
India – 560 064
bharathish@revainstitution.org

Vidyasagar K N
Asst. Professor, Dept. of ECE
Reva ITM, Bangalore-64
India – 560 064
vidyasagarkn@revainstitution.org

Abstract

In this paper an efforts has been made to estimate pixel intensity based on information present in the whole image and thereby exploiting the presence of similar patterns and features. The method is called non-local image de-noising method. This estimates noise-free pixel intensity as a weighted average of all pixel intensities in the image, and the weights are proportional to the similarity between the local neighborhood of the pixel being processed and local neighborhoods of surrounding pixels. This non-local method works on the assumption that image contains an extensive amount of redundancy. But we can find many pixels which have very less number of pixels with similar neighborhood in an image, especially at edges and when the image is corrupted with higher noise variance. Because of this, de-noised image visual quality is less. For further improvement in visual quality of image at the edges KL transform based local filter is designed and applied.

Keywords— KL Transform; de-noising; non-local method; visual quality.

INTRODUCTION

Basically there are two types of image de-noising methods such as Local means algorithms and Non-Local means algorithms. In first case, to restore the intensity of particular pixel only the local neighborhood of pixel being processed is used whereas in second case the entire image is taken into account to restore the intensity of particular pixel. Former case makes assumption about the frequency content of the image whereas later case assumes that image contains an extensive amount of redundancy, which means to say that the image has large number of similar patterns. The assumption made by non-local method works very well for images which are having no much grey level transitions or the edges. For pixels that fall near the edges of the image, there are less numbers of pixels with the similar neighborhoods. Because of this reason we see blurred de-noised image near the edges. To avoid this blurring effect we are introducing a local post processing filer which is based on KL transform. This post processing method is more effective for images that are corrupted by higher noise variances, for example image corrupted by random noise with noise variance, $\sigma = 30$. In this paper a non-local means algorithm is implemented for standard database images such as Lena image, Barbara image, Baboon image, Couple image etc.
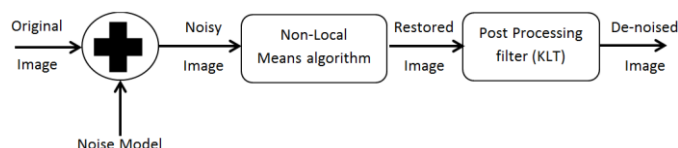

Fig 1: Block diagram of Image De-noising and Post Processing.

This paper is organized as follows: Section I gives brief introduction, section II deals with basic Non-Local means algorithm and noise model, which generate Additive White Gaussian Noise (AWGN) (uncorrelated), section III deals with implementation of post processing filter, its pseudo code, section IV is about results and discussions while conclusion is given in section V.

BASIC NON-LOCAL ALGORITHM

A. Basic Non-Local Means Algorithm

The self-similarity assumption can be de-noised an image. Pixels with similar neighborhoods can be used to determine the de-noised value of pixel. Weights are assigned to pixels on the basis of their similarity with the pixel being reconstructed. While assessing the similarity, the pixel under consideration as well as neighborhood is taken into account. Mathematically, it can be expressed as:

$$NL[u](x) = \frac{1}{C(x)} \int_\Omega e^{-\frac{(G_a * |u(x+.)-u(y+.)|^2)(0)}{h^2}} u(y)dy \tag{1}$$

The integration is carried out over all the pixels in the search window. Where

$$C(x) = \int_{\Omega} e^{-\frac{(G_a * |u(x+.) - u(z+.)|^2)(0)}{h^2}} dz \qquad (2)$$

$C(x)$ is a normalized constant. G_a a Gaussian kernel and h is a filtering parameter [2].

B. Noise Model [5]

This model generates Additive White Gaussian Noise (AWGN) for different noise variances and this noise will be added to the original standard image.

II. POST PROCESSING FILTER, ITS PSUEDO CODE

A. Post Processing Filter [3]

This is local filter, which is based Karhunen Loeve transform, applied on blocks of reconstructed image where there is higher noise variance to improve the visual quality of the image. This KL transform shows best de-correlating capability with uncorrelated noise and zero means and minimizes the mean square error (MSE).The KL transform was originally introduced as series expansion for continuous random processes by Karhunen and Karhunen Loeve. For random sequences Hoteling first studied what was called a method of principal components, which is the discrete equivalent of the KL series expansion. Consequently, the KL transform is also called the Hotelling transform or the method of principal components.

Properties of KL transform:

The KL transform has many desirable properties, which make it optimal in many signal processing applications. Here we have discussed two important properties for de-noising an image. Let an $N \times N$ image $u(m,n)$ is represented by a random field. For simplicity we assume u has zero mean and a positive definite covariance matrix R.

➢ **Decorrelation:** The KL transform coefficients $\{v(k), k = 0,\ldots\ldots,N-1\}$ are uncorrelated and have zero mean, that is

$$E[v(k)] = 0 \qquad (3)$$

$$E[v(k)v^*(l)] = \lambda_k \delta(k - l) \qquad (4)$$

The proof follows directly from above equations, since

$$E[vv^T] \triangleq \Phi^{*T}E[uu^T]\Phi = \Phi^{*T}R\Phi = \Lambda \qquad (5)$$

It should be noted that Φ is not a unique matrix with respect to this property. There could be many matrices that would de-correlate the transformed sequence.

➢ **Basic restriction mean square error:** Consider the operations in fig 2. The vector u is first transformed to v. The elements of w are chosen to be the first m elements of v and zero elsewhere. Finally, w is transformed to z. A, B are $N \times N$ matrices and I_m is a matrix with a 1s along the first m diagonal terms and zeros elsewhere. Hence

$$w(k) = \begin{cases} v(k), & 0 \le k \le m - 1 \\ 0, & k \ge m \end{cases} \qquad (6)$$

Therefore, whereas u and v are vectors in an N-dimensional vector space, w is a vector restricted to an $m \le N$ - dimensional subspace. The average mean square error between the sequence $u(n)$ and $z(n)$ is defined as

$$J_m \triangleq \frac{1}{N} E\left(\sum_{n=0}^{N-1} |u(n) - z(n)|^2\right)$$

$$= \frac{1}{N} Tr[E\{(u - z)(u - z)^{*T}\}] \qquad (7)$$

This quantity is called the *basis restriction error*. It is desired to find the matrices A and B such that J_m is minimized for each and every value of $m \in [1,N]$. This minimum is achieved by the KL transform of u.
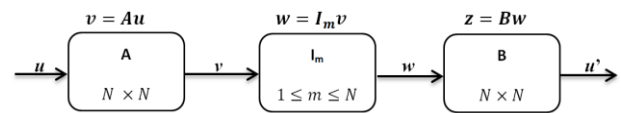

Fig 2. Image Post Processing Using KL transform

Theorem: The error J_m is minimum when

$$A = \Phi^{*T}, \qquad B = \Phi, \qquad AB = I$$

Where, the columns of Φ are arranged according to the decreasing order of the eigenvalues of R.

B. Psuedo Code for Post Processing Filter

Following steps should be repeated throughout the restored image, i.e. for all blocks of the image.

Step1. Take a window u of size $N \times N$.

Step2. Calculate the *noisevariance*.

Step3. If the *noisevarieance > cut-off* (depends on the image, for lena image its around 20)

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for every row vector of u, calculate mean as

$$m = \frac{1}{N} \sum_{i=1}^{N-1} u_i^\rho$$

Calculate covariance matrix R_u as

$$R_u = \frac{1}{(N-1)} \sum_{i=0}^{N-1} (u_i^\rho - \mu^\rho)(u_i^\rho - \mu^\rho)^T$$

Calculate eigenvalues, λ_i, and orthonormal vectors, Φ, of R_u.

Rearrange the orthonormal vectors according to decreasing order of their eigenvalues

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Step4. Compute KL transform as

$$\bar{v}_n = \Phi^T u_n, \qquad 1 \le n \le N - 1$$

Step5. Retain first $N/2$ rows (it can be varied) of v_n and set others to zero, then obtain invers KL transform as

$$u'_n = \Phi v_n, \qquad 1 \le n \le N - 1$$

Replace u by u'

Results and Discussions

The proposed algorithm is implemented in MATLAB R2010a and is tested on PC, Pentium(R) dual core CPU, 2GB RAM. Original standard test images are added with Additive White Gaussian Noise generated by noise model and processed with proposed de-noising algorithm.

The functional parameters used are:
  i.   Noise variance, Sigma = 50
  ii.   Search window size = 5 X 5
  iii. KLT block size = 16 X 16

Fig 1(a-d) shows the images corrupted by Additive White Gaussian Noise with PSNR value 14.14. Non-Local means estimated, with improved PSNR, and post processed or filtered, with improved visual quality, de-noised versions of noise corrupted images are also displayed.
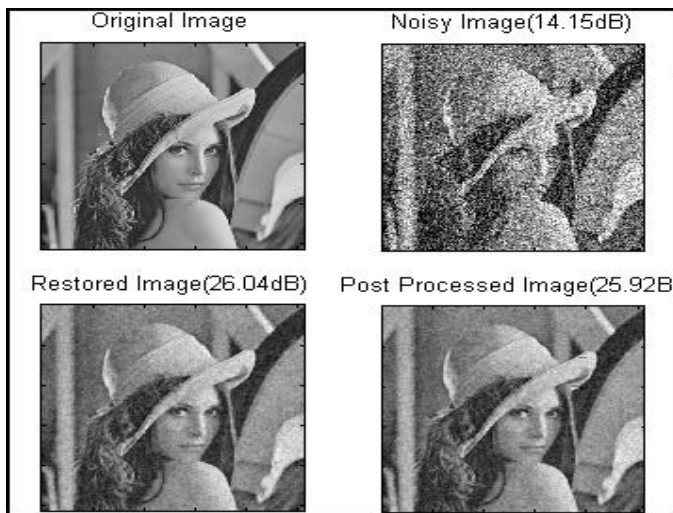

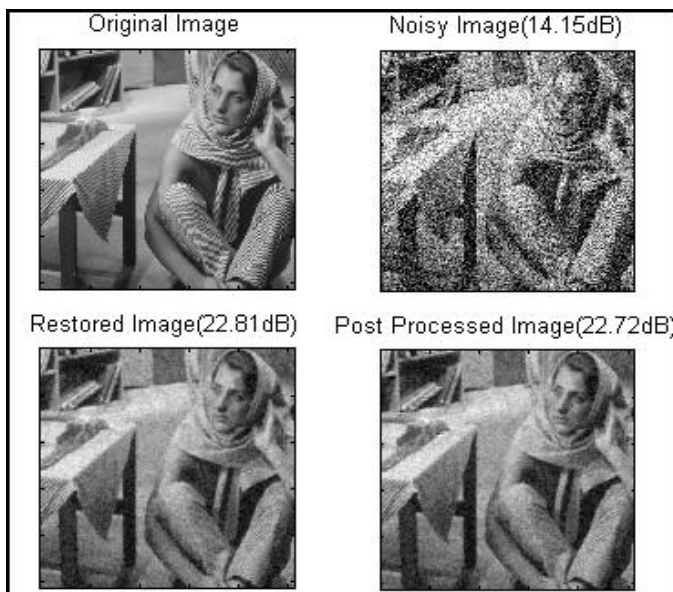*Fig1 (a).Lena image added with AWGN, sigma = 50*

Fig1 (b).Barbara image added with AWGN, sigma = 50

Table I. Summarizes the MSE computed with respect to the Original Image for different images with additive Gaussian noise type, restored image, post processed image respectively.

TABLE I.

TABLE II.   COMPARISON OF MEAN SQUARE ERROR(MSE) RESULTS.

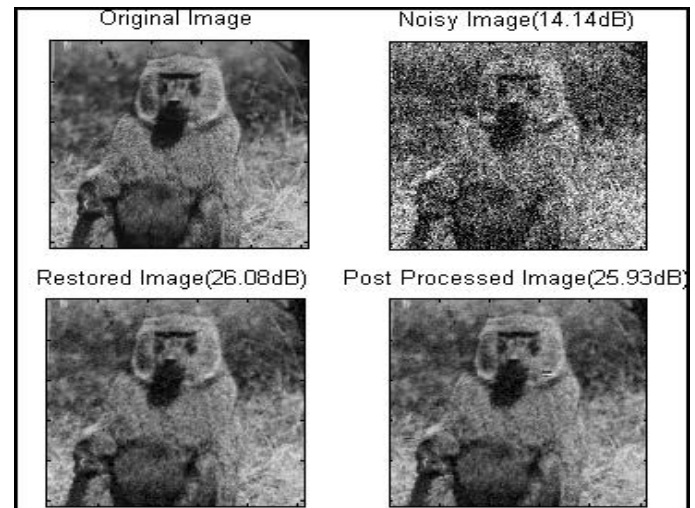| Standard Images	Proposed Algorithm, MSE						Basic Non-Local Means Algorithm[2], MSE for σ = 20
	Before Post Processing			After Post Processing			
	Noise variance, σ						
	20	25	50	20	25	50	
Lena	**49**	65	159	**58**	71	160	**68**
Baboon	**62**	75	159	**63**	77	162	**292**
Barbara	127	172	340	147	177	342	
Couple	106	132	246	121	154	262	

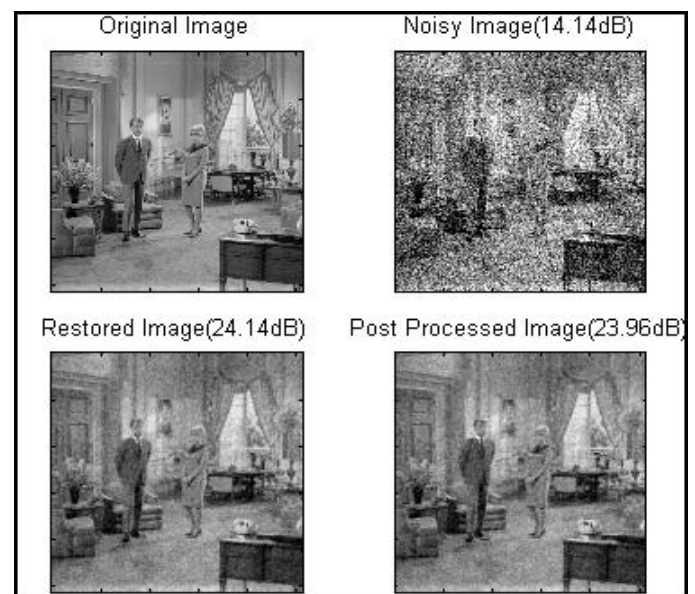*Fig1 (c).Baboon image added with AWGN, sigma = 50*

Fig1 (d).Couple image added with AWGN, sigma = 50

CONCLUSION

From the fig 1(a-d), we can see that the PSNR is decreased from restored image to post processed image, though the PSNR is decreased we can see the improvement in the visual quality of post processed image, especially at edges. It is know that lesser the MSE means that estimate is nearer to the original one, but this is not the case always. From the table1 it is proved that the results are still comparable with the Basic

Non-Local means algorithm [2]. For images corrupted by noise of variance 50, we observe only smaller change in MSE compared to the images corrupted by lesser noise variances. Thus, proposed algorithm works better for images that are corrupted by higher noise variances.

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