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Investigation of Transducer Mass Loading Effect in Frequency Response Function (FRF)

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Abstract:
The accuracy and the reliability of various analyses using the measured FRFs depend strongly on the quality of measured data. It is well known that the quality of measured frequency response functions (FRFs) is adversely affected by many factors, most significant sources being noise and systematic errors like mass loading effects of transducers. A transducer mounted on a vibrating system changes the dynamics of the structure due to the addition of extra mass and introduces errors into measured FRFs. One problem with this is the production of unrealistic results, which cause the measured resonant frequencies to be less than the correct values. These errors also lead to incorrect prediction of modal parameters. In many situations, the mass loading effect is ignored in the analytical and experimental Modelling process, based on a usual assumption that the transducer mass is negligible compared to that of the structure under test. However, when light-weighted structures are investigated, this effect can be significant and it may be necessary to eliminate this undesirable side effect before the measured data are used for further analyses.

The mass loading effects of accelerometer and force transducer can be eliminated from measured FRFs (including point FRF and transfer FRF) in shaker modal testing. Considering different sensors for response measurements, two common collocations in shaker modal testing are investigated: (1) shaker + Laser Doppler vibrometer case, in which only force transducer mass loading effects need to be removed, and (2) shaker + accelerometer case, in which both accelerometer and force transducer mass should be eliminated.

The Sherman-Morrison identity for the elimination of mass loading effects of accelerometers from measured FRF. The formulation presented can be applied for both fixed transducer (hammer testing) and moving transducer (shaker testing) case. In moving transducer case, a dummy mass is utilized. Also, the transducer mass loading effect on the transfer FRFs can be removed by considering a set of measurements using two accelerometers with different masses. Sometimes, the effect of the extra masses on a measured FRF can be cancelled by cancellation technique for transducers at the driving points.

The resonance frequencies of the plate measured with an accelerometer are lower than those of measured without accelerometer. However, after the elimination of the effect of the mass difference between the two accelerometers, both the natural frequencies and the FRFs as a whole are in quite good agreement with the target values.
Keywords: Modal Analysis, Frequency Response Function, modal parameters, mass loading effects, etc..

1. Introduction

This Frequency response functions (FRFs) measurement is an importance process in modal testing. The quality of FRFs measured on a structure has been a concern of vibration engineers for a considerable period of time. Accurate FRFs measurement is the prerequisite to obtain high-precision modal parameters. However, the measured FRFs are often inaccurate due to various factors in the testing process. Among these, one of the unavoidable error sources is the so-called mass loading effects of transducers.

In modal testing, some sensors (such as force transducer and accelerometer) have to be mounted on the test structure. The dynamics of the test structure are therefore changed and the measured FRFs contain errors consequently, such as deviation of the measured resonant frequencies from their correct values. It is desirable in practice that these deviations are acceptably small as they may cause considerable difficulties in many applications depending on the level of errors induced by transducer mass loading during measurement.

For large structure under test, the mass loading effects are ignored based on a usual assumption that the transducer mass is negligible compared to that of the structure. However, as the mass of the transducer approaches that of the test article i.e. the test structure is small and lightweight, this effect can be significant. Lightweight structures are those structures that optimize the load carrying capacity of the elements by large deflection, allowing the load to be taken primarily in tension. It is characterized by having small mass relative to the applied load which the shape of the structure is determined through an optimization process. Lightweight structures include cable, membrane, shell, thin plate and folded structures. In such cases, it is necessary to eliminate this undesirable side effect beforethe measured data are used for further analysis

2. Frequency Response Function

These functions are used in vibration analysis and modal testing. There are many tools available for performing vibration analysis and testing. The frequency response function is a particular tool. A frequency response function (FRF) is a transfer function, expressed in the frequency domain. Frequency response functions are complex functions, with real and imaginary components. They may also be represented in terms of magnitude and phase. A frequency response function can be formed from either measured data or analytical functions.

Consider a linear system as represented by the diagram in Fig.1. F(ω) is the input force as a function of the angular frequency w . H(ω) is the transfer function. X(ω) is the displacement response function. Each function is a complex function, which may also be represented in terms of magnitude and phase.

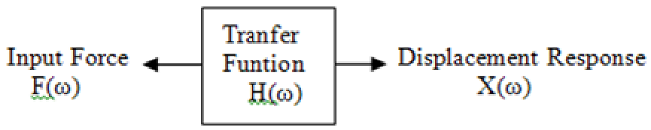

Figure 1: Frequency Response Function (FRF)

3. Theory of Mass Loading

The mass of an accelerometer can significantly affect the dynamic characteristics of the structure to which it is mounted. This is commonly called mass loading effect which tends to lower the measured natural frequencies. The general rules is the accelerometer mass should be less than one-tenth from the effective mass of the structure to which it is attached. Theoretically, the natural frequency is ;

$$\omega = \sqrt{(K/M)}$$

The addition of the accelerometer mass to the mass of the vibrating structure changes the resonant frequency of the vibrating systems as follows;

$$f_m = f_s \sqrt{K/(M+ma)}$$

Where ω = natural frequency
K = stiffness of the structure
M = mass of the structure
Ma = accelerometer mass
f_m= frequency of the structure with the influence of the accelerometer mass
f_s = frequency of the structure without the influence of the accelerometer mass

This relationship shows that if the accelerometer mass is kept small compared to the mass of the structure then any changes in the vibration will be only small. The mass loading produced by accelerometer depends on the local dynamic properties of the structure. The mass and resulting frequencies shift is proportional to the square of deflection of the associatedmode. This study will determine how much the natural frequency will change due to the mass loading effect.

4. Calculation of Natural Frequencies

Consider the cantilever beam with and without accelero- meter mounted on it. The beam data is as follows:

Dimension	: 300 x 46.5 x 5.1 mm
Density :	: 7850 kg/m3
Modulus of elasticity	: 210 GPa
Mass of Cantilever Beam	: 610.8 gm
Mass of accelerometer	: 27.5 gm

4.1 Analytical Method

If the cross-sectional dimensions of beam are small as compared to its length, the system is known as Euler-Bernoulli Beam. Only thin beams are treated under this category. Euler-Bernouli equation is used for calculation of natural frequencies of beam with and without accelerometer mounted on it.

1) *Natural Frequency of Beam w/o accelerometer*

Bernouli equation for cantilever beam is as follows,

$$\frac{d^2}{dx^2}\left(EI\frac{\mathrm{dw}^2}{\mathrm{dx}^2}\right) + \rho A\frac{\mathrm{dw}^2}{\mathrm{dx}^2} = 0$$

Where,

W(x) = Acoshβx + Bsinhβx + Ccosβx + Dsinβx

The natural frequencies can be calculated as,

$$\omega_i = (\beta_i l)^2\sqrt{(EI/\rho A l^4)}$$

2) *Natural Frequency of Beam with accelerometer*

By Considering the various boundary conditions at fixed and free end and solving it, we obtained the following equation,

$$1 + cos\beta l cosh\beta + \mu\beta l(\sinh\beta l\cos\beta l - \cosh\beta l\sin\beta l) = 0$$

Where,

$$\mu = \frac{\mathrm{M}}{\rho \mathrm{Al}} = \frac{\text{Mass of accelerometer } (m_a)}{\text{Mass of beam} (m_b)}$$

4.2 Finite Element Method

Using FEM we will find the natural frequencies of the continuous cantilever beam. The Basic procedure is outlined here

1) In the first step, the geometry is divided into a number of small elements. The elements may be of different shapes and sizes.
2) Then elemental equations are obtained for each element.
3) In the third step the elemental equations are assembled to yield a system of global equation.
4) The problem is solved by reduced down to the equation,
$\{[M]\omega 2 + [K]\} X = 0$

The above equation represents the standard Eigen value problem whose solution gives Eigen vectors and Eigen values. The Eigen values represent the square of the natural frequencies and the Eigen vectors represent the corresponding mode shapes.

In MATLAB the cantilever beam is generated with different number of elements until the previous result and current result do have negligible difference. The result is shown in table 4.1 below. From the result it is shown that beyond 10 elements the changes are insignificant.

Therefore for all further simulation 10 element beams is considered.

TABLE 1: Result Of FE Model With Different Number Of Elements

No. of Elements	*ω1*	*% Change w.r.t. Previous Value*	*ω2*	*% Change w.r.t. Previous Value*	*ω3*	*% Change w.r.t. Previous Value*
3	297.51	--	1870.4	--	5285.1	--
5	297.48	--	1865.2	0.275	5238.8	0.875
7	297.48	--	1864.5	0.039	5225.4	0.256
10	297.48	--	1864.3	0.0076	5221.4	0.076
12	297.48	--	1864.3	0.0003	5220.7	0.013

4.3 Validation with Euler-Bernoulli Equation

In this value of natural frequencies of 10 element beam with and without accelerometer is compared with analytical values.

TABLE 2: Validation of FE Model

Natural frequency (rad/sec)	*Without Accelerometer*		*With Accelerometer*	
	Analytical	*FEM*	*Analytical*	*FEM*
ω1	297.4832	297.4835	271.799	271.794
ω2	1864.2960	1864.357	1724.66	1724.683
ω3	5220.0825	5221.412	4872.986	4873.953

4.4 Accelerometer Mass Loading Error

The natural frequencies of FE beam with and without accelerometer are compared below:

TABLE 3: Accelerometer Mass Loading Error

Natural frequency(rad/sec)	*ω1*	*ω2*	*ω3*
Without accelerometer mass	297.4835	1864.357	5221.412
With accelerometer mass	271.794	1724.683	4873.923
% change	8.633	7.49	6.649

5. Mass Loading Cancellation Technique

In shaker modal testing, force transducer mounted on the excitation point is to measure the excitation signal from which we obtain the excitation force applied to the structure. However, the measured force deviates from the exact force applied to the structure due to the force transducer mass loading effects.

Fig. 2 shows a modal test of a cantilever beam. The shaker excited the structure at point p through the force transducer which aims to obtain the exact force fexa. However, the actual measured force is fmeas which is different from fexa due to the mass loading effects.

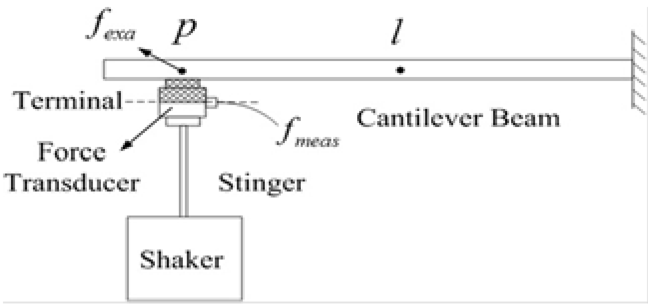

Figure 2: Shaker modal test of a cantilever beam

To obtain the effective force or exact force f_{exa} applied to the beam, one must subtract from the measured force fmeas, the inertia force corresponding to the total extra mass mf,

$$f_{exa} = f_{meas} - m_f \times a_p$$

$$f_{exa}(\omega) = f_{meas}(\omega) - m_f \times a_p(\omega)$$

1) Point FRF:The force transducer and the accelerometer are mounted at the same point when the point FRF is measured i.e. input and output is measured at same locations. The two transducers masses can be considered as a concentrated mass which can be eliminated from the measured FRFs simultaneously.

2) Transfer FRF: The force transducer and the accelerometer are mounted at different points when transfers FRFs are measured. i.e. input and output is measured at different locations.

Dividing ap(ω) by both sides of above Eq., we get the relationship between App and A(p)pp ,

$$A_{pp} = \frac{a_p(\omega)}{f_{exa}(\omega)} = \frac{a_p(\omega)/f_{meas}(\omega)}{1-m_f a_p(\omega)/f_{meas}(\omega)} = \frac{A_{pp}^{(p)}}{1-m_f A_{pp}^{(p)}}$$

Where A(p)pp is measured point FRF relating p, App is exact point FRF relating p. Similarly, dividing al(ω) by both sides of Eq. , we get the relationship between Alp and A(p)lp .

5.1 Shaker + laser Doppler vibrometer

Laser Doppler vibrometer is welcome in shaker modal testing for its high-precision advantage, especially the very advantage of non-contact measurement that will not introduce extra mechanical effects to the test. Therefore, only force transducer mass loading effects need to be eliminate in this case.

However, we can only get the velocity FRFs from the testing since the Laser Doppler vibrometer is to measure velocity signal. This poses no problem as accelerance and velocity FRF are just two different forms of presenting the same FRF. Despite the advantage of non-contact measurement, Laser Doppler vibrometer brings a high costing.

5.2 Shaker + accelerometer

Shaker + accelerometer test program is another universal case in shaker modal testing. Different from the Laser, accelerometer will introduce extra mass loading to the test structure. The correction becomes more complicated as both force transducer and accelerometer mass effects should be eliminated.

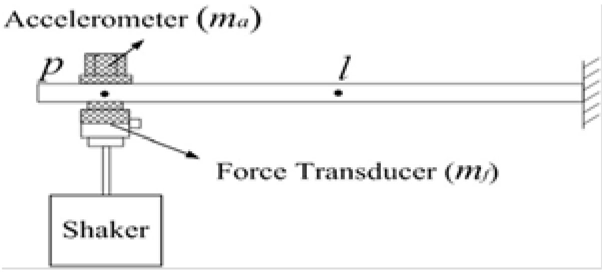

Figure 3: The measurement of point FRFA(p1 ,p2)$_{pp}$.

Fortunately, the force transducer and the accelerometer are mounted at the same point when the point FRF is measured, as is shown in Fig. 3.The corresponding correction formula can be easily obtained by replacing mf with (mf + ma).

The force transducer and the accelerometer are mounted at different points when transfers FRFs are measured. Therefore, it is essentially a problem of eliminating multi-d.o.f mass loading effects from measured transfer FRFs. Elimination can be done step by step. Accelerometer mass loading effects will be eliminated in the first step, while, the force transducer in the second step by using the method mentioned before. The accelerometer mass loading effects can be eliminated from transfer FRFs by two methods namely,

1) Using two accelerometers with different masses
2) Using an accelerometer and a dummy mass

6 Verification of the Method

Fig. 4 shows that the resonance frequencies of the beam measured with a accelerometer are lower than those of measured without a accelerometer. However, after the elimination of the effect of the mass loading of accelerometer, both the natural frequencies and the FRFs as a whole are in quite good agreement with the target values. Similar observations can also be made for point FRF illustrated in Figs.5

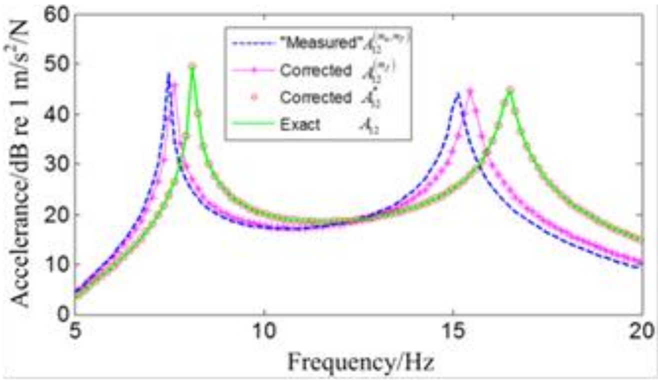

Figure 4: Comparison of measured, corrected and exact transfer FRF A_{12}

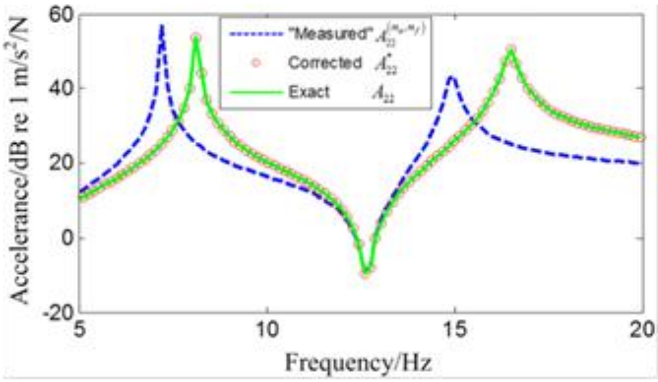

Figure 5: Comparison of measured, corrected and exact transfer FRFA$_{22}$

7 Assessment of Quality Of The Measurement

From a practical point of view, comparison between the natural frequencies of the measured FRFs and those of the exact FRFs is a suitable way of assessing the quality of the measurement. When there is no discernible change in the natural frequency of the structure due to the attachment of an accelerometer, the measured FRFs can be judged to have a good quality; otherwise, the results are not reliable and there is a need to cancel the mass loading effect of the accelerometer.

If the driving point FRF at the point of the attachment of the accelerometer, $A^{(l)}{}_{ll}$, could be measured then the exact driving point FRF,A_{ll} , would be computed .A comparison between$A^{(l)}{}_{ll}$and A_{ll} shows the difference between the natural frequencies of the structure and those of the modified structure. However, sometimes measurement of$A^{(l)}{}_{ll}$ is not practical. In these cases the following method is suggested for obtaining the exact natural frequencies of the structure:

If an extra mass is added to the accelerometer and the structure is measured again, the exact FRF can be obtained using the method,

$$\begin{Bmatrix} A_{li} \\ A_{ll} \end{Bmatrix} = \left(\begin{bmatrix} \frac{1}{A_{li}^{(l)}} & -m_1 \\ \frac{1}{\bar{A}_{li}^{(l)}} & -m_2 \end{bmatrix} \right)^{-1} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

in which A_{li} and A_{ll} are exact FRFs; $A^{(l)}_{li}$ and $\bar{A}^{(l)}_{li}$ are two measurements using two different masses for the accelerometer.
The frequencies that make the determinant of the matrix equal to zero are the exact natural frequencies of the structure. This means that the exact natural frequencies of the structure are at the frequencies where:

$$\frac{A_{li}^{(l)}}{\bar{A}_{li}^{(l)}} = \frac{m_2}{m_1}$$

If the magnitude of m_2 is twice that of m_1, then we have,

$$A^{(l)}_{li} = 2\,\bar{A}^{(l)}_{li}$$

This means that if $\bar{A}^{(l)}_{li}$ is doubled and drawn on a graph together with $A^{(l)}_{li}$, the point of intersection of 2 $\bar{A}^{(l)}_{li}$ and $A^{(l)}_{li}$ represents the exact natural frequency of the structure. Fig. 6 shows the graphical representation of this method. Therefore the exact natural frequency of the structure can be estimated either by the addition to the structure of an extra mass, with the same mass as that of the accelerometer, and using the graphical method shown in Fig. 6. If, by doubling the mass of the accelerometer, a major difference is not discerned between $A^{(l)}_{li}$ and $\bar{A}^{(l)}_{li}$, it can be concluded that the natural frequency of the structure has not been changed by the addition of the accelerometer. In this case the quality of the original measurement is acceptable.

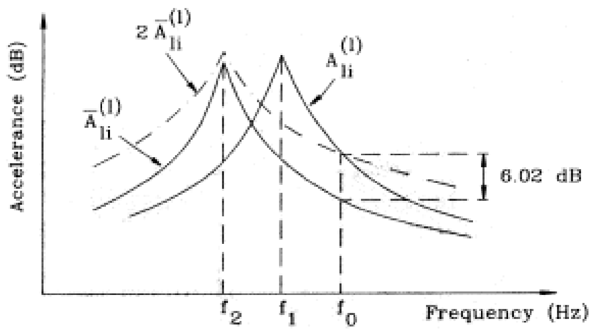

Figure 6: Estimation of the changes in the natural frequency of the structure by doubling the mass of the accelerometer

f_0 = Natural frequency of the structure
f_1 = Natural frequency of the structure and the accelerometer
f_2 = Natural frequency of the structure and the accelerometer and an extra mass with the same mass as that of the accelerometer.

8 Excitation Technique

Excitation Systems introduce unwanted forces and moments at the excitation location. Unfortunately, it is neither easy nor feasible to measure those forces and moments for each FRF measurements. There are two larger groups of excitation systems, Impact Excitation System and Shaker Excitation System.

8.1 Impact Excitation System

When no parts of excitation system are fixed to measure the structure, there is no unintentional loading on the structure. It is non-contact type excitation system. Impacting the structure with hammer is an often used excitation method. The force input of hammer on structure is measured with force cell i.e. connected to the hammer. With hammer excitation, no force cell is fixed to the structure. This means that there is no loading of structure due to the fixation of excitation equipment on input location. It is used in fixed Transducer case.

When output is fixed and FRF are measured for multiple inputs, this corresponds to measuring elements from single row of FRF matrix.

8.2 Shaker Excitation System

With contact excitation systems, the connection between shaker and test structure loads the structure. This can be very critical when mass of excitation system is not negligible w.r.t. mass of tested structure. In most application the excitation system is supposed to deliver unidirectional external force along measurement axis of force transducer. This assumption is not exact. Forces and moments in all direction need to be considered. These loads do not depend on characteristics of shaker only, they also depend on type and magnitude of deformation the structure is expected to exhibit. It is used in moving Transducer case.

When input is fixed and FRF are measured for multiple outputs, this corresponds to measuring elements from single column of FRF matrix.

9 Conclusions

The resonance frequencies of the beam measure with an accelerometer are lower than those of measured without an accelerometer. This is because of mass loading effect of accelerometer. However, after elimination of mass loading effect of accelerometer both natural frequencies and FRFs as a whole are in quite good agreement with target values. This accelerometer mass loading effect is eliminated from both Fixed FRF and Transfer FRF.

In case of Shaker + Laser Doppler Vibrometer, only force transducer mass need to be eliminated, while both accelerometer and force transducer mass should be removed in case of Shaker + Accelerometer. The quality of measurement relating to the mass loading effect of accelerometer can be assessed by estimating the exact natural frequencies of the structure.

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