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(τ_i, τ_j)-ρ-Continuous Maps in Biopological Spaces

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Abstract.
In this paper, to introduce (τ_i, τ_j)- ρ –continuous maps from a bitopological space (X,τ_1, τ_2) into a bitopological space (Y, σ_1, σ_2) and study some of their Properties.
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1. Introduction

In 1963, J.C. Kelly[14] defined a bitopological space (X,τ_1, τ_2) to be a set X equipped with two topologies τ_1, τ_2 on X and he initiated a systematic study of bitopological space. The study of generalized closed sets in a bitopological space was initiated by Levine in [15] and the concept of $T_{1/2}$ spaces was introduced. Various authors, like I. Arockiarani [2], S.P. Arya and T.M. Nour [3], R. Devi [8] and Y. Gnanambal [12] and have turned attention to the various concepts of topology by considering bitopological spaces instead of topological spaces. In 1996, H.Maki, J. Umehara and T. Noiri [16] introduced the classes of pregeneralized closed sets and used them to obtain properties of pre-$T_{1/2}$ spaces. The modified forms of generalized closed sets and generalized continuity were studied by K. Balachandran, P. Sundaram and H. Maki [4]. In 2008, S. Jafari, T. Noiri, N. Rajesh and M.L. Thivagar [13] introduced the concept of $\tilde{g}$-closed sets and their properties. In 2014, O. Uma Maheswari, A. Vadivel and D. Sivakumar[19] introduced the concept of (τ_i, τ_j)-$\#rg$-closed sets and their properties. In 2016, O. Uma maheswari introduced $(\tau i, \tau j) - \rho$ -closed sets in bitopological spaces.

In this paper, to introduce new classes of continuous functions called (τ_i, τ_j)-$\rho - \sigma_k$ –continuous functions in bitopological spaces. During this process, some of their properties are obtained. And also to introduce the concept of $\rho -$ bicontinuous, $\rho -$ strongly bi continuous and pairwise $\rho -$ irresolute in bitopological spaces.

Before entering into our work we recall the following definitions, which are due to various authors.

2. Preliminaries

Throughout this paper (X,τ_1, τ_2), $(Y, \sigma_1, \sigma_2)$and (Z, η_1, η_2) will always denote bitopological spaces on which no separation axioms are assumed, unless otherwise mentioned. When A is a subset of (X,τ_1, τ_2), $Cl(A)$, $Int(A)$ and $D[A]$ denote the closure, the interior and the derived set of A, respectively.

Definition 2.1. Let a subset A of a space (X, τ) is called
1. Regular open [18] if $A= \tau_i\text{-}int(\tau_j\text{-} cl(A))$ *and regular closed if* $A= \tau_j\text{-}cl(\tau_i\text{-}int(A))$.
2. π *-open [22] if it is the finite union of* (τ_i, τ_j)*- regular open sets.*
3. Regular semiopen [6] if there is a regular open set U such that $U \subseteq A \subseteq cl(U)$.

Definition 2.2. *Let* (X,τ_1, τ_2) *be a bitopological space and* $A\subseteq X$.

1. (τ_i, τ_j)-*Preopen [9] if* $A \subseteq \tau_i$*-int*$(\tau_j$*-cl*$(A))$ *and preclosed if* τ_j*-cl*$(\tau_i$*-int*$(A)) \subseteq A$.
2. (τ_i, τ_j)-*Semi-open [9] if* $A \subseteq \tau_j$*-cl*$(\tau_i$*-int*$(A))$ *and semi-closed if* τ_i*-int*$(\tau_j$*-cl*$(A)) \subseteq A$.
3. (τ_i, τ_j)- α*-open [9] if* $A \subseteq \tau_i$*-int*$(\tau_j$*-cl*$(\tau_i$*-int*$(A)))$ *and* α *-closed if* τ_j*-cl*$(\tau_i$*-int*$(\tau_j$*-cl*$(A))) \subseteq A$.
4. (τ_i, τ_j)- *Semi preopen [9] if* $A \subseteq \tau_j$*-cl* $(\tau_i$*–int* $(\tau_j$*–cl* $(A)))$ *and semi preclosed if* τ_i*–int* $(\tau_j$*-cl* $(\tau_i$*-int*$(A))) \subseteq A$.

The Pre-interior of A, denoted by pint(A), is the union of all preopen subsets of A.
The Pre-closure of A, denoted by Pcl(A), is the intersection of all Preclosed sets containing A.

Definition 2.5. *Let* (X, τ_1, τ_2) *be a topological space. A subset* $A \subseteq X$ *is said to be*

1. (τ_i, τ_j)-*generalized closed (briefly g-closed)[10] if* τ_j*-cl*$(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is* τ_i*-open in X.*
2. (τ_i, τ_j)-*generalized preclosed (briefly gp-closed)[11] if* τ_j*-Pcl*$(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is* τ_i*- open in X.*
3. (τ_i, τ_j)-*generalized preregular closed (briefly gpr-closed)[12] if* τ_j*-Pcl*$(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is* τ_i*-regular open in X.*
4. (τ_i, τ_j)-*pregeneralized closed (briefly pg-closed)[11] if* τ_j*-Pcl*$(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is* τ_i*-preopen in X.*
5. (τ_i, τ_j)-*g*-preclosed (briefly g*p-closed)[11] if* τ_j*-Pcl*$(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is* τ_i*-g-open in X.*
6. (τ_i, τ_j)-*generalized semi-preclosed (briefly gsp-closed)[9] if* τ_j*-spcl*$(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is* τ_i*-open in X.*
7. (τ_i, τ_j)- π*gp-closed [17] if* τ_j*-Pcl*$(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is* τ_i*-*π*-open in X.*
8. (τ_i, τ_j)- *rw closed [5] if* τ_j*-cl*$(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is* τ_i*–regular semi open in X*
9. (τ_i, τ_j)- *#rg- closed [19] if* τ_j*-cl*$(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is* τ_i*-rw-open in X*

Definition 2.4. *A map* $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ *is called*
(i) $\tau_j - \sigma_k$ *-continuous [10] if* $f^{-1}(V) \in \tau_j$ *, for every* $V \in \sigma_k$.
(ii) $\tau_j - \sigma_k$ *-semi-continuous [15] if* $f^{-1}(V) \in \tau_j$ semiclosed *, for every* $V \in \sigma_k$.
(iii) $\tau_j - \sigma_k$ *-* α *-continuous [15] if* $f^{-1}(V) \in \tau_j - \alpha closed$ *, for every* $V \in \sigma_k$.
(iv) τ_j *-* σ_k *-pre-continuous [15] if* $f^{-1}(V) \in \tau_j$*- preclosed , for every* $V \in \sigma_k$.
(v) τ_j *-* σ_k *-semi-pre-continuous [15] if* $f^{-1}(V) \in \tau_j$ *–semi-preclosed , for every* $V \in \sigma_k$.
(vi) D(i, j) - σ_k *-continuous [10] if* $f^{-1}(V) \in D(i, j)$ *for every* σ_k *-closed set in* (Y, σ_1, σ_2) .
(vii) Dgs(i, j) - σ_k *-continuous [15] if* $f^{-1}(V) \in Dgs(i, j)$ *for every* σ_k *–closed set in* (Y, σ_1, σ_2) .
(viii) Dgsp(i, j) - σ_k *-continuous [15] if* $f^{-1}(V) \in Dgsp(i, j)$ *for every* σ_k *- closed set in* (Y, σ_1, σ_2)
(ix) Dr(i, j) - σ_k *-continuous [1] if* $f^{-1}(V) \in Dr(i, j)$ *for every* σ_k *-closed set in* (Y, σ_1, σ_2) .

Definition 2.5. *A map* $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ *is called*
(i) bi-continuous [10] if f is $\tau_1 - \sigma_1$ *-continuous and* $\tau_2 - \sigma_2$ *-continuous.*
(ii) regular generalized bi-continuous (rg -bi-continuous) [1] if f is Dr(τ_1, τ_2) *-* σ_2 *-continuous and Dr*(τ_2, τ_1) *-* σ_1 *-continuous.*
(iii) generalized semi pre-bi-continuous (gsp -bi-continuous) [15] if f is Dgsp(τ_1, τ_2) *-* σ_2 *-continuous and Dgsp*$(\tau_2, \tau_1) - \sigma_1$ *-continuous.*

Definition 2.6. *A function* $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ *is called*
(i) strongly bi-continuous (s -bi-continuous) [10] if f is bi -continuous, $\tau_1 - \sigma_2$ *-continuous and* $\tau_2 - \sigma_1$ *-continuous.*
(ii) regular generalized strongly bi-continuous (rg - s -bi-continuous) [1] if f is g -bi-continuous, D$(\tau_1, \tau_2) - \sigma_1$ *-continuous and D*$(\tau_2, \tau_1) - \sigma_2$ *-continuous.*
(iii) generalized semi pre-strongly bi-continuous (gsp - s -bi-continuous) [15] if f is gsp -bi-continuous, Dgsp$(\tau_2, \tau_1) - \sigma_2$ *-continuous and Dgsp*$(\tau_1, \tau_2) - \sigma_1$ *-continuous.*

3 (τ_i, τ_j)-ρ--Continuous functions in bitopological spaces

Definition 3.1. A map f : (X, τ_1, τ_2) ⟶ (Y, σ_1, σ_2) is called Dρ(τ_i, τ_j) - σ_k -continuous if the inverse image of every σ_k -closed set in (Y, σ_1, σ_2) is an (τ_i, τ_j)- ρ-closed set in (X, τ_1, τ_2) .

Remark 3.2. Suppose that $\tau_1 = \tau_2 = \tau$ and $\sigma_1 = \sigma_2 = \sigma$ in Definition 3.1., then the D_ρ (τ_i, τ_j) - σ_k -continuity of maps coincides with ρ –continuity [14] of maps in topological spaces.

Theorem 3.3. If a map f : (X, τ_1, τ_2) ⟶ (Y, σ_1, σ_2) is τ_j - σ_k -continuous, then it is a D_ρ (τ_i, τ_j) - σ_k -continuous map.
Proof: Let V be a σ_k -closed set in (Y, σ_1, σ_2) . Then f^{-1}(V) is τ_j -closed set.
By Theorem 3.1 in [16], so f^{-1}(V) is (τ_i, τ_j) - ρ -closed set in (X, τ_1, τ_2) .
Therefore f is D_ρ (τ_i, τ_j) - σ_k -continuous.
The converse of the above theorem need not be true as seen from the following example.

Example 3.4. Let X = {a, b, c} , τ_1 = {X, ∅, {a}, {a, c}} , τ_2 = {X, ∅, {a}, {b}, {a, b}} and Y = {p, q} , σ_1 = {Y, ∅, {p}} and σ_2 = {Y, ∅, {q}} . Define a map f : (X, τ_1, τ_2) ⟶ (Y, σ_1, σ_2) by f(a) = f(c) = p and f(b) = q . Then f is D_ρ (τ_1, τ_2) – σ_2 -continuous but f is not τ_1 - σ_2 - continuous.

Theorem 3.5. If a map f : (X, τ_1, τ_2) ⟶ (Y, σ_1, σ_2) is τ_j - σ_k – pre continuous, then it is a D_ρ (τ_i, τ_j) - σ_k -continuous map.
Proof: Let V be a σ_k -closed set in (Y, σ_1, σ_2) . Then f^{-1}(V) is τ_j – pre closed set.
By Theorem 3.1 in [16], so f^{-1}(V) is (τ_i, τ_j) - ρ -closed set in (X, τ_1, τ_2) .
Therefore f is D_ρ (τ_i, τ_j) - σ_k -continuous.
The converse of the above theorem need not be true as seen from the following example.

Example 3.6. Let X = {a, b, c} , τ_1 = {X, ∅, {a}, {a, c}} , τ_2 = {X, ∅, {a}, {b}, {a, b}} and Y = {p, q} , σ_1 = {Y, ∅, {p}} and σ_2 = {Y, ∅, {q}} . Define a map f : (X, τ_1, τ_2) ⟶ (Y, σ_1, σ_2) by f(a) = f(c) = p and f(b) = q . Then f is D_ρ (τ_1, τ_2) – σ_2 -continuous but f is not τ_1 - σ_2 - pre continuous.

Theorem 3.7. If a map f : (X, τ_1, τ_2) ⟶ (Y, σ_1, σ_2) is D_ρ (τ_i, τ_j) - σ_k -continuous map, then it is a D_{gp} (τ_i, τ_j) - σ_k -continuous map.
Proof: Let V be a σ_k -closed set in (Y, σ_1, σ_2) . Then f^{-1}(V) is τ_j – ρ closed set.
By Theorem 3.1 in [16], so f^{-1}(V) is (τ_i, τ_j) - gp -closed set in (X, τ_1, τ_2) .
Therefore f is D_{gp} (τ_i, τ_j) - σ_k -continuous.
The converse of the above theorem need not be true as seen from the following example.

Example 3.8. *Let X* = {*a*, *b*, *c*, *d*} , τ_1= { ϕ, {*c*}, {*a*, *b*}, {*a*, *b*, *c*}, *X*} and τ_2 = { ϕ, {b, *c*}, {*a*, *b*, *c*}, {b, c, d}, *X*} . Y = {p, q} , σ_1 = {Y, ∅, {p}} and σ_2 = {Y, ∅, {q}} . Define a map f : (X, τ_1, τ_2) ⟶ (Y, σ_1, σ_2) by f(a) = p and f(b) = f(c) = f(d) = q. *Then f is* D_{gp} (τ_i, τ_j) - σ_k *-continuous but not* D_ρ (τ_i, τ_j) - σ_k *-continuous in* (*X*,τ_1, τ_2).

Theorem 3.9. If a map f : (X, τ_1, τ_2) ⟶ (Y, σ_1, σ_2) is D_ρ (τ_i, τ_j) - σ_k -continuous map, then it is a D_{gpr} (τ_i, τ_j) - σ_k -continuous map.

Proof: Let f : (X, τ_1, τ_2) ⟶ (Y, σ_1, σ_2) is D_ρ (τ_i, τ_j) - σ_k -continuous map.
Let G be any σ_k -closed set in (Y, σ_1, σ_2). Then f^{-1}(G) is (τ_i, τ_j) - ρ – closed set in (X, τ_1, τ_2).
And by Theorem 3.3 in [16], f^{-1}(G) is (τ_i, τ_j) - gpr – closed set in (X, τ_1, τ_2) .
Hence f is D_{gpr} (τ_i, τ_j) - σ_k -continuous map.

The converse of the above theorem need not be true as seen from the following example.

Example 3.10. In *example 3.4,*
(τ_i, τ_j) *-ρ –closed set :* { ϕ, {b}, {c}, {a, b}, {a, c}, {b, c }, X}, (τ_i, τ_j) -gpr *closed set :P(X),*
Then the set A = {a} *is* (τ_i, τ_j)*- gpr closed but not* (τ_i, τ_j)*- ρ -closed in* (X, τ_1, τ_2). *Then f is* a D_{gpr} (τ_i, τ_j) - σ_k -continuous map but not D_ρ (τ_i, τ_j) - σ_k -continuous map.

Theorem 3.11. If a map f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is D_ρ (τ_i, τ_j) - σ_k -continuous map, then it is a D_{gsp} (τ_i, τ_j) - σ_k -continuous map.
Proof: Let f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is D_ρ (τ_i, τ_j) - σ_k -continuous map.
Let G be any σ_k -closed set in (Y, σ_1, σ_2). Then $f^{-1}(G)$ is (τ_i, τ_j) - ρ – closed set in (X, τ_1, τ_2).
And by Theorem 3.3 in [16], $f^{-1}(G)$ is (τ_i, τ_j) - gsp – closed set in (X, τ_1, τ_2) .
Hence f is D_{gsp} (τ_i, τ_j) - σ_k -continuous map.
The converse of the above theorem need not be true as seen from the following example.

Example 3.12. In Example 3.8,
The set A= {a, c} *is* (τ_i, τ_j) *-gsp- closed but not* (τ_i, τ_j) *-ρ –closed in* (X, τ_1, τ_2).*Then f is* D_{gsp} (τ_i, τ_j) - σ_k -continuous map but not D_ρ (τ_i, τ_j) - σ_k -continuous map.

Theorem 3.13. If a map f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is D_ρ (τ_i, τ_j) - σ_k -continuous map, then it is a $D_{\pi gp}$ (τ_i, τ_j) - σ_k -continuous map.
Proof: Let f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is D_ρ (τ_i, τ_j) - σ_k -continuous map.
Let G be any σ_k -closed set in (Y, σ_1, σ_2). Then $f^{-1}(G)$ is (τ_i, τ_j) - ρ – closed set in (X, τ_1, τ_2).
And by Theorem 3.3 in [16], $f^{-1}(G)$ is (τ_i, τ_j) - πgp – closed set in (X, τ_1, τ_2) .
Hence f is $D_{\pi gp}$ (τ_i, τ_j) - σ_k -continuous map.
The converse of the above theorem need not be true as seen from the following example.

Example 3.14. *Let X* = {a, b, c} *and* τ_1 = { ϕ, {a}, {a, c}, X}, τ_2 = { ϕ, {a}, {b}, {a, b}, X}.
Y = {p, q} , σ_1 = {Y, ∅, {p}} and σ_2 = {Y, ∅, {q}} . Define a map f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ by f(a) = p and f(b) = f(c) = f(d) = q.
The set A = {a} *is* (τ_i, τ_j)*- πgp-closed but not* (τ_i, τ_j)*-ρ -closed in* (X, τ_1, τ_2). Then f is $D_{\pi gp}$ (τ_i, τ_j) - σ_k -continuous map but not D_ρ (τ_i, τ_j) - σ_k -continuous map.

Remark 3.15. (τ_i, τ_j)*-ρ-*σ_k*-continuous and* (τ_i, τ_j)*-* σ_k *pre continuous are independent concepts as we illustrate by means of the following examples.*

Example 3.16.
As in Example 3.7, the set A= {a, b} *is* (τ_i, τ_j)*- ρ - closed but not* (τ_i, τ_j)*- Preclosed in* (X, τ_1, τ_2)*, then f is* (τ_i, τ_j)*-ρ-*σ_k *-continuous but not* (τ_i, τ_j)*-* σ_k *pre continuous*

Remark 3.17. (τ_i, τ_j)*-ρ-* σ_k *-continuous are independent concepts of* (τ_i, τ_j)*-* σ_k*semi- continuous and* (τ_i, τ_j)*-* σ_k*Semi-Pre continuous as we illustrate by means of the following example.*

Example 3.18. *Let X* = {a, b, c, d} *and* τ_1 = { ϕ, {b}, {c}, {b, c}, {b, c, d}, X} and τ_2 = *{ϕ, {a}, {b}, {a, b}, X}*. Y = {p, q} , σ_1 = {Y, ∅, {p}} and σ_2 = {Y, ∅, {q}} .
Define a map f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ by f(a) = f(b) = f(c) = p and f(d) = q
(τ_i, τ_j)*-ρ closed set :*{ϕ, {a},{a, b},{a, c}, {a, d},{b, d}, {c, d},{a, b, c}, {a, b, d},{a, c, d},{b, c, d}, X},
(τ_i, τ_j) –Semi *closed set :* { ϕ, {c}, {d}, {a, c}, {b, c}, {c, d}, {a, c, d}, {b, c, d}, X},
(τ_i, τ_j) –Semi pre *closed set :* { ϕ, {a},{c}, {d}, {a, b}, {a, c}, {a, d}, {c, d}, {a, c, d}, {b, c, d}, X}

Then the set A = {a, b, c} is (τ_i, τ_j)- ρ -closed but neither (τ_i, τ_j)- semi-closed nor (τ_i, τ_j)- semi-pre-closed then (τ_i, τ_j)-ρ- σ_2-continuous but neither (τ_i, τ_j)- σ_2 −semi- continuous and (τ_i, τ_j)- σ_2 − Semi-Pre continuous.

Remark 3.20. *(τ_i, τ_j)- ρ-σ_k -continuous and (τ_i, τ_j)- g*p- σ_k- continuous are independent concepts as we illustrate by means of the following example.*

Example 3.21.
Let X = {a, b, c, d, e}. Let τ_1 = { ϕ, {a, b}, {a, b, d}, {a, b, c, d}, {a, b, d, e, }, X} and τ_2 = { ϕ, {b}, {d, e}, {b, d, e}, {a, c, d, e, }, X}. Y = {p, q, r} , σ_1 = {Y, ∅, {p}, {r}, {p,r}} and σ_2 = {Y, ∅, {q}}. Define a map f : (X, τ_1, τ_2) ⟶ (Y, σ_1, σ_2) by f(b) = f(c) = f(d) =p, f(e)= r and f(a) = q
*The set B = {a} is (τ_i, τ_j)- g*p-closed but not (τ_i, τ_j)- ρ –closed in (X, τ_1, τ_2). Then (τ_i, τ_j)- ρ-σ_k -continuous and (τ_i, τ_j)- g*p- σ_k- continuous are independent*

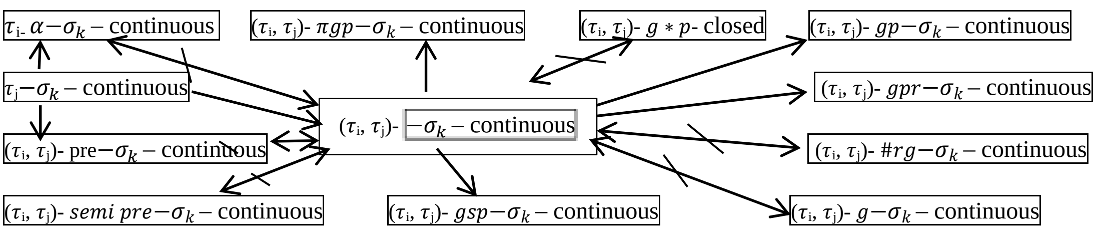
Remark 3.22. *(τ_i, τ_j)-ρ- σ_k -continuous and (τ_i, τ_j)-α-σ_k-continuous are independent concepts as we illustrate by means of the following examples.*

Example 3.23. *As in Example 3.4*
The set A= {a, b} is (τ_i, τ_j)- ρ -closed but not (τ_i, τ_j)- α -closed in (X,τ_1, τ_2). Then (τ_i, τ_j)-ρ- σ_k -continuous and (τ_i, τ_j)-α-σ_k-continuous are independent

Remark 3.24. *(τ_i, τ_j)- ρ- σ_k -continuous and (τ_i, τ_j)- # rg- σ_k-continuous are independent concepts as we illustrate by means of the following examples.*

Example 3.25.
Let X = {a, b, c}, τ_1= { ϕ, {a}, {b}, {a, b}, X}, τ_2= { ϕ, {a}, {a, c}, X}. Let Y = {p, q} ,
σ_1 = {Y, ∅, {p}} and σ_2 = {Y, ∅, {q}} . Define a map f : (X, τ_1, τ_2) ⟶ (Y, σ_1, σ_2) by f(a) = f(b) = q and f(c) = p. *Then set A= {c} is (τ_i, τ_j)- ρ -closed but not (τ_i, τ_j)- # rg -closed in (X,τ_1, τ_2).*
Then (τ_i, τ_j)- ρ- σ_k -continuous and (τ_i, τ_j)- # rg- σ_k- continuous are independent.

Remark 3.29. *From the above discussions and known results should be accompanied by a reference we have the following implications A⟶ B (A=B) represents A implies B but not conversely (A and B are independent of each other →. See Figure 1.* \
Figure 1: Implications.



4 Some stronger forms of ρ –continuous functions in bitopological spaces

Definition 4.1. *A map f : (X, τ_1, τ_2) ⟶ (Y, σ_1, σ_2) is called*
(i) ρ -bi-continuous if f is Dρ(τ_1, τ_2) - σ_2 -continuous and Dρ(τ_2, τ_1) - σ_1-continuous.
(ii) ρ-strongly-bi-continuous (briefly ρ- s -bi-continuous) if f is ρ-bi-continuous, Dρ (τ_1, τ_2)- σ_1 continuous and Dρ (τ_2, τ_1) -σ_2-continuous.

Theorem 4.2. *Let f : (X, τ_1, τ_2) ⟶ (Y, σ_1, σ_2) be a map.*

(i) If f is bi-continuous, then f is ρ -bi-continuous.
(ii) If f is s -bi-continuous, then f is ρ - s -bi-continuous.
Proof (i) Let f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *be a bi-continuous. Then f is* τ_1 - σ_1*-continuous and* τ_2 - σ_2 *-continuous. Since every* τ_j - σ_k *-continuous map is* $D\rho(\tau_i, \tau_j)$ - σ_k *-continuous. If follows that f is* $D\rho(\tau_1, \tau_2)$ - σ_2 *–continuous and* $D\rho(\tau_2, \tau_1)$ - σ_1 *-continuous. Thus f is* ρ *-bi-continuous.*
(ii) Let f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *be a s -bi-continuous. Then f is bi-continuous and* τ_1 - σ_2 *-continuous and* τ_2 - σ_1 *-continuous. By (i), it follows that f is* $D\rho(\tau_1, \tau_2)$- σ_2 *-continuous,* $D\rho(\tau_2, \tau_1)$- σ_1 *-continuous and* ρ *bi-continuous. Thus f is* ρ *- s -bi-continuous.*
The converse of the above theorem need not be true as seen from the following example.

Example 4.3. *Let* $X = \{a, b, c\}$, $\tau_1 = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$, $\tau_2 = \{X, \emptyset, \{a\}, \{b, c\}\}$ *and*
$Y = \{p, q\}$, $\sigma_1 = \{Y, \emptyset, \{q\}\}$ *and* $\sigma_2 = \{Y, \emptyset, \{p\}\}$.
Define a map f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *by* $f(a) = q$ *and* $f(b) = f(c) = p$.
Then the map f is ρ *- s -bi-continuous but not s -bi-continuous. This map is also* ρ *-bi-continuous but not bi-continuous.*

Theorem 4.4. *Let f :* $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *be a map.*
(i) If f is ρ *-bi-continuous, then f is gsp -bi-continuous.*
(ii) If f is ρ *- s -bi-continuous, then f is gsp - s -bi-continuous.*
Proof (i) Let f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *be a* ρ *-bi-continuous. Then f is* $D\rho(\tau_1, \tau_2)$ - σ_2*-continuous and* $D\rho(\tau_2, \tau_1)$ - σ_1*-continuous. By Theorem 3.3., f is* $Dgsp(\tau_1, \tau_2)$- σ_2*-continuous and* $Dgsp(\tau_2, \tau_1)$ - σ_1*-continuous. Thus f is gsp -bi-continuous.*
(ii) Let f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *be* ρ *- s -bi-continuous. Then f is* $D\rho(\tau_1, \tau_2)$ - σ_2 *-continuous,* $D\rho(\tau_2, \tau_1)$ - σ_1 *-continuous and* ρ *-s -bi-continuous, By (i), it follows that f is* $Dr(\tau_1, \tau_2)$ - σ_2 *-continuous and* $Dr(\tau_2, \tau_1)$ - σ_1 *-continuous and gsp -bi-continuous. Thus f is gsp - s –bicontinuous map.*
The converse of the above theorem need not be true as seen from the following example.

Example 4.5. *In the Example 4.2, Define a map f :* $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *by* $f(a) = f(b) = q$ *and* $f(c) = p$ *. The map f is gsp - s -bi-continuous but not* ρ *- s -bi-continuous. This map is also gsp -bi-continuous but not* ρ *- bi-continuous.*

Definition 4.6. *A map f :* $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is called pairwise* ρ *-irresolute map if* $f^{-1}(A) \in D\rho(\tau_i, \tau_j)$ *in* (X, τ_1, τ_2) *for every* $A \in D\rho(\sigma_k, \sigma_e)$ *in* (Y, σ_1, σ_2) .

Theorem 4.7. *If f:* $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is pairwise* ρ *–irresolute then f is* $D\rho(\tau_i, \tau_j)$ - σ_k *-continuous.*
Proof: Let F be any σ_k *-closed set in Y . Then f is* (σ_k, σ_e) *-*ρ *-closed set.*
Since every τ *-closed set is* (τ_i, τ_j) - ρ *-closed set. And so* $F \in D\rho(\sigma_k, \sigma_e)$.
Since f is pairwise ρ *-irresolute,* $f^{-1}(F) \in D\rho$- (τ_i, τ_j) *. Therefore f is* $D\rho(\tau_i, \tau_j)$ - σ_k *-continuous.*
The converse of the above theorem need not be true as seen from the following example.

Example 4.8. *Let* $X = Y = \{a, b, c\}$, $\tau_1 = \{X, \emptyset, \{a\}\}$, $\tau_2 = \{X, \emptyset, \{a\}, \{a, c\}\}$ *and* $\sigma_1 = \{Y, \emptyset, \{a\}, \{b\}, \{a, b\}\}$, $\sigma_2 = \{Y, \emptyset, \{a, c\}\}$ *. Define a map f :* $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *by* $f(a) = c$, $f(b) = b$ *and* $f(c) = a$ *. Then f is* $D\rho(\tau_1, \tau_2)$ - σ_2*-continuous but f is not a pairwise* ρ *-irresolute map.*

Theorem 4.6. *If f :* $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *and g :* $(Y, \sigma_1, \sigma_2) \to (Z, \eta_1, \eta_2)$ *are pairwise* ρ *-irresolute maps, then their composition gof is also pairwise* ρ *-irresolute.*
Proof: Let $A \in D\#rg(\eta_m, \eta_n)$ *in* (Z, η_1, η_2)*. Since g is pairwise* ρ *-irresolute,* $g^{-1}(A) \in D\rho(\sigma_k, \sigma_e)$ *in* (Y, σ_1, σ_2) *. Again by hypothesis,* $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A) \in D\rho(\tau_i, \tau_j)$ *in* (X, τ_1, τ_2) .

And so gof is pairwise ρ -irresolute map.

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