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Investigation on Three-dimensional viscoelastic nanofluid with Newtonian heating and binary chemical reaction

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Abstract
This paper investigated the steady three dimensional boundary layer flow of an incompressible viscoelastic nanofluid in the presence of Newtonian heating and chemical reaction. A mathematical formulation has designed for momentum, temperature and nanofluid solid volume profiles. The coupled partial differential equations are solved using the Nactsheim-Swigert shooting technique with the six order Runge-Kutta iteration Method. The results for the dimensionless velocity, temperature, and nanofluid solid volume profiles are discussed with the help of graphs.

1. Introduction

A dilute suspension made up of nanometer-sized particles and fibres dispersed in a liquid is known as nanofluid . Accordingly, their physical properties such as; velocity, density, thermal and electrical conductivities are superior as compared with those of the base fluids. The most important of the physical properties of nanofluids, is thermal conductivity owing to its many applications. The conventional fluids such as water, oil and ethylene glycol mixtures exhibit poor thermal conductivity and therefore are not very suitable for heat transfer. Their application as cooling tools can increase manufacturing and operating costs. To enhance the thermal conductivity of these fluids, nanoparticles are suspended in these liquids. Nanofluids are made of ultrafine nanoparticles of the order of <100nm suspended in a base fluid such as water or an organic solvent. Nanofluids are found to exhibit higher conductive and convective heat transfer performances as compared to the conventional fluids

The subject of non –Newtonian fluids have been a popular and important area of researchers for last many more. Comparing with viscous fluids the Mathematical models of non-Newtonian fluids are more complex in nature and of higher order Despite of all these challenges, Many researchers have given attention to valuable contributions of variety of non-Newtonian fluids. (1-6)amongst these, viscoelastic fluids are considered to be more important in the present are due to its wide range engineering and industrial manufacturing applications. Some of these materials have shear-independent viscosity but most of them have shear rate viscosity. Some of the existing approaches in this direction include Hayat et al.(7) wrok in which they discussed thermal radiation effects in three dimensional mixed convection flow of viscoelastic fluid. The steady, laminar boundary flow and heat transfer with radiation effects using the nonlinear Rosseland approximation induced in a inactive, electrically conducting, visco-elastic fluid studied by Cortell(8).Shehzad et al.(9) explained the magneto hydrodynamic(MHD) boundary layer flow past a stretching sheet of three dimensional viscoelastic fluid in the presence of thermal radiation and variable thermal conductivity. Alhuthali et al.(10) investigated three dimensional viscoelastic fluid flow past an exponentially stretching sheet with mass transfer.

In general certainty in nanofluids, the base fluids do not aligned with the characteristics of Newtonian fluids. So it becomes more justified to think of them as viscoelastic fluids, examples may include Ethylene glycol- Al_2o_3 Ethylene glycol- CuO and Ethylene glycol- ZnO , as viscoelastic nanofluids.Many research works can be done on two and three dimensional viscoelastic fluids but very less literature have been studied in case of viscoelastic nanofluids, especially , three dimensional viscoelastic nanofluids. Choi(11) pioneering work has opened the gates for successors to explore the new dimensions in this avenue. The effects of viscous dissipation and Newtonian heating on third grade nanofluid illustrated by Shehzad et al.(12) and Khan et al.(13) investigated the influence of heat generation/absorption on boundary layer three-dimensional flow of an Oldroyd-B nanofluid towards a stretching surface. Boundary layer flow past a bi-directional exponentially stretching surface of nanofluid with convective boundary conditions is studied by Khan et al(14) Hayat et al.(15) dissussed the mixed convection flow of viscoelastic fluid by a Stretching cylinder with heat transfer. Qayyum et al.(16) presented the Newtonian heating effects in three dimensional flow of viscoelastic fluid. Stagnation-point flow of a nanofluid towards a stretching sheet was discussed by Mustafa et al.(17) Khan and Pop(18) studied the boundary –layer flow of a nanofluid past a stretching sheet. Makinde and aziz(19) explained the boundary layer flow of a nanofluid past a stretching sheet with a convective boundary condition. Boundary layer flow of nanofluid over exponentially stretching surface was examined by Nadeem and Lee.(20) Mustafa et al.(21) discussed the Numerical and solutions for stagnation-point flow of a nanofluid over an exponentially stretching sheet.

Homotopy analysis method (HAM) (22-27) has been used to tackle of three-dimensional viscoelastic nanofluid past a bi-directional stretching sheet related problem. Ramzan and Farhan yousaf (28) investigated Boundary layer flow of three-dimensional viscoelastic nanofluid past a bi-directional stretching sheet with Newtonian heating. We have extended the investigation of Ramzan and Farhan yousaf (28) with binay chemical reaction and activation energy. A mathematical formulation has designed for momentum, temperature and nanofluid solid volume profiles. The coupled partial differential equations are solved using the Nactsheim-Swigert shooting technique with the six order Runge-Kutta iteration Method. The results for the dimensionless velocity, temperature, and nanofluid solid volume profiles are discussed with the help of graphs.

1.1 Mathematical formulation

Consider three dimensional flow of an incompressible viscoelatic nanofluid pasr a bi-directionally stretching sheet located at z=0 in a linear manner with Newtonian heating and binary chemical rection. Let u,v,w be the velocities in the x,y,z directions respectively.

The governing equations of conservation of mass, momentum,energy and concentration are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=\nu\frac{\partial^2 u}{\partial z^2}-k_1\left[\begin{array}{l} u\frac{\partial^3 u}{\partial x\partial z^2}+w\frac{\partial^3 u}{\partial z^3} \\ -\left(\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial z^2}+\frac{\partial u}{\partial z}\frac{\partial^2 w}{\partial z^2}+2\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial x\partial z}+2\frac{\partial w}{\partial z}\frac{\partial^2 u}{\partial z^2}\right)\end{array}\right], \qquad (2)$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=\nu\frac{\partial^2 v}{\partial z^2}-k_1\left[\begin{array}{l} v\frac{\partial^3 v}{\partial y\partial z^2}+w\frac{\partial^3 v}{\partial z^3} \\ -\left(\frac{\partial v}{\partial y}\frac{\partial^2 v}{\partial z^2}+\frac{\partial v}{\partial z}\frac{\partial^2 w}{\partial z^2}+2\frac{\partial v}{\partial z}\frac{\partial^2 v}{\partial y\partial z}+2\frac{\partial w}{\partial z}\frac{\partial^2 v}{\partial z^2}\right)\end{array}\right], \qquad (3)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\alpha_m\frac{\partial^2 T}{\partial z^2}+\tau\left[D_B\frac{\partial C}{\partial z}\frac{\partial T}{\partial z}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial z}\right)^2\right] \qquad (4)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}+w\frac{\partial C}{\partial z}=D_B\frac{\partial^2 C}{\partial z^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial z^2}-K_r^2(C-C_\infty)\left(\frac{T}{T_\infty}\right)^n e^{\left(-\frac{E_a}{\kappa T}\right)} \qquad (5)$$

Where ν , k_1 ,T ,C and α_m are the kinematic viscosity , material parameter of fluid, temperature, nanoparticle's concentration and is the thermal diffusivity respectively. The term $K_r^2(C-C_\infty)\left(\frac{T}{T_\infty}\right)^n e^{\left(-\frac{E_a}{\kappa T}\right)}$ in equation (4) represents the modified Arrhenius equation in which K_r^2 is the reaction rate, E_a the activation energy, κ = 8.61x10^{-5}eV/K the Boltzmann constant and n fitted rate constant which generally lies in the range -1 < n < 1.

The corresponding boundary conditions are

$$\begin{array}{l} u=u_w(x)=ax,\ v=v_w(y)=by, \\ w=0, \frac{\partial T}{\partial z}=-h_s T,\ \frac{\partial C}{\partial z}=-h_s C\ \ at\ \ z=0, \\ u\to 0,\ v\to 0,\ \frac{\partial u}{\partial z}\to 0,\ \frac{\partial v}{\partial z}\to 0, \\ T\to T_\infty,\ C\to C_\infty\ \ as\ \ z\to\infty \end{array} \qquad (6)$$

Where the bi-directional stretching velocities u = ax along the x-axis and v = by along the y-axis, h_s is the heat transfer coefficient and T_∞ and C_∞ are the temperature and nanoparticle concentration away from the surface, D_B is the Brownian motion coefficient, D_T is the thermophoretic diffusion coefficient and $\tau = (\rho c)_p / (\rho c)_f$ is the ratio of effective heat capacity of the nanoparticle material to heat capacity of the fluid.

By using similarity transformations, we have

$$\eta = \sqrt{\frac{a}{\nu}} z,\ u = axf'(\eta),\ v = ayg'(\eta),$$
$$w = -\sqrt{a\nu}\{ f(\eta) + g(\eta)\},$$
$$\theta(\eta) = \frac{T - T_\infty}{T_\infty},\quad \phi(\eta) = \frac{C - C_\infty}{C_\infty}, \tag{7}$$

The equations (1)-(5) can be written in the following manner:

$$f''' - f'^2 + (f+g)f'' + K\left[(f+g)f^{(iv)} + (f''-g'')f'' - 2(f'+g')f'''\right] = 0 \tag{8}$$

$$g''' - g'^2 + (f+g)g'' + K\left[(f+g)g^{(iv)} + (g''-f'')g'' - 2(f'+g')g'''\right] = 0 \tag{9}$$

$$\theta'' + \Pr(f+g)\theta' + Nb\theta'\phi' + Nt\theta'^2 = 0 \tag{10}$$

$$\phi'' + \Pr Le(f+g)\phi' + \frac{Nt}{Nb}\theta'' - Le\sigma(1+\delta\theta)^n \varphi e^{\left(-\frac{E}{1+\delta\theta}\right)} = 0 \tag{11}$$

$$f(0) = 0,\ g(0) = 0\ \ f'(0) = 1,\ g'(0) = s,$$
$$\theta'(0) = -\gamma\left[1+\theta(0)\right],\ \phi'(0) = -\gamma\left[1+\phi(0)\right],$$
$$f'(\infty) = 0,\ g'(\infty) = 0,\ f''(\infty) = 0,$$
$$g''(\infty) = 0,\ \theta(\infty) \to 0,\ \phi(\infty) \to 0, \tag{12}$$

Where K, s, γ, Pr, Le, Nb, Nt, E, δ, and σ are the viscoelastic parameter, ratio parameter, conjugate parameter for Newtonian heating, Prandtl number, Lewis number, Brownian motion parameter, thermophoresis parameter, non-dimensional energy (E),temperature difference parameter (δ), dimensionless reaction rate (σ) and fitted rate constant (*n*) respectively. These are given by

$$K = \frac{k_1 a}{\nu},\ s = \frac{b}{a},\ \gamma = \sqrt{\frac{\nu}{a}} h_s,\ \Pr = \frac{\nu}{\alpha_m}, Le = \frac{\alpha_m}{D_B},$$
$$\sigma = \frac{K_r^2}{c}\quad \delta = \frac{T_w - T\infty}{T\infty}\quad E = \frac{E_a}{\kappa T} \tag{13}$$

1.2 Results and discussion

In this section provide a clear insight of the problem, the velocity, temperature and concentration profiles have been analyzed by assigning numerical values to the governing dimensionless parameters such as Ratio parameter (s), Viscoelastic parameter (K), Newtonian heating parameter (γ),Prandtl number (Pr),Lewis number (Le), Brownian motion Parameter (Nb),Thermophoresisparameter (Nt),non-dimensional energy (E),temperature difference parameter (δ), dimensionless reaction rate (σ) and fitted rate constant (*n)* respectively. Numerical computations are shown graphically from figures.1-15.

Fig.1 & Fig.2 shows that the variation of Ratio parameter (s) on the velocity components f′ and g′.From this figures illustrate that velocity component f′ decrease and g′ increase with an increasing values of Ratio parameter (s).

Fig.3, Fig.4, Fig.5 and Fig.6 shows that the variation of Viscoelastic parameter (K) on the velocity, temperature and concentration profiles. From this figures illustrate that velocitycomponents f′ and g′ decreases with an increasing values of Viscoelastic parameter (K). But temperature and concentration profiles both increases with an increasing values of Viscoelastic parameter (K).

Fig.7 & Fig.8 shows that the variation of Newtonian heating parameter (γ)on the temperature and concentration profiles.From this figures illustrate both profiles enhance with an increasing values of Newtonian heating parameter (γ).

Fig.9 & Fig.10 shows that the variation of Prandtl number (Pr)on the temperature and concentration profiles. From this figures illustrate both profiles are decreases with an increasing valuesofPrandtl number (Pr).

Fig.11 & Fig.12 shows that the variation of Brownian motion parameter(Nb) and temperature difference parameter (δ) on the concentration profile. From this figures illustrate concentration profile decreases with an increasing values of Brownian motion parameter(Nb) and temperature difference parameter (δ).

Fig.13 & Fig.14 shows that the variation of Thermophoresis parameter (Nt) and dimensionless reaction rate (σ) on the concentration profile. From this figures illustrate concentration profile enhance with an increasing values of Thermophoresis parameter (Nt) decreases with an increasing values of dimensionless reaction rate (σ).

Fig.15 shows that the variation of non-dimensional energy (E) on the concentration profile. From this figures illustrate concentration profile enhance with an increasing values of non-dimensional energy (E).

Fig 1: Influence of s on f'

Fig 5: Influence of K on θ

Fig 2: Influence of s on g'

Fig 6: Influence of K on ¢

Fig 3: Influence of K on f'

Fig 7: Influence of γ on θ

Fig 4: Influence of K on g'

Fig 8: Influence of γ on ϕ

Fig 9: Influence of Pr on θ

Fig 10: Influence of Pr on ϕ

Fig 11: Influence of Nb on ϕ

Fig 12: Influence of δ on ϕ

Fig 13: Influence of Nt on ϕ

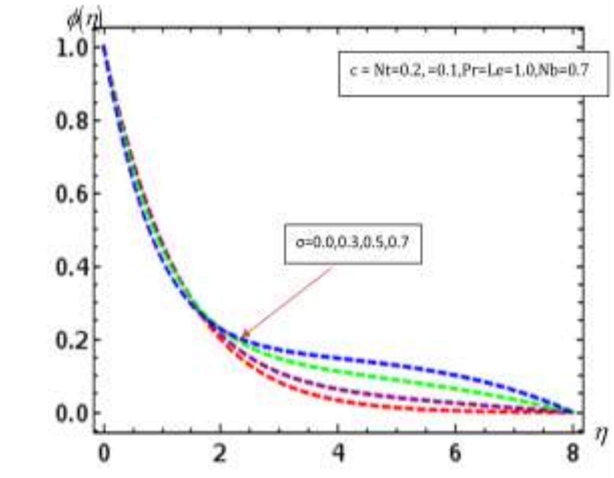

Fig 14: Influence of σ on ϕ

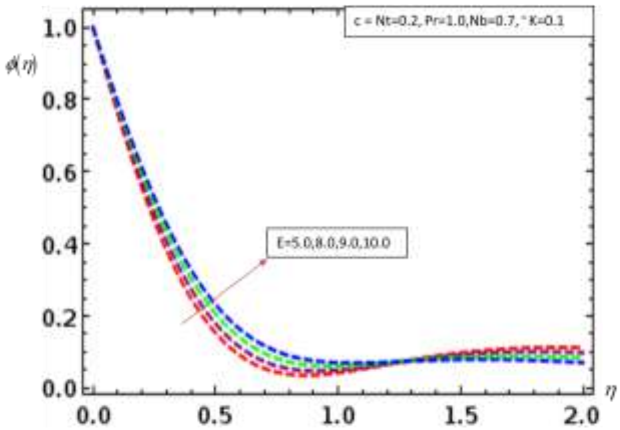

Fig 15: Influence of E on ϕ

2. Conclusions

In this investigation, Boundary layer flow of three-dimensional viscoelastic nanofluid past a bi-directional stretching sheet with Newtonian heating with binay chemical reaction and activation energy. The velocity, temperature, and concentration profiles analyzed by assigning numerical values to the governing dimensionless parameters such as Ratio parameter (s), Viscoelastic parameter (K), Newtonian heating parameter (γ),Prandtl number (Pr),Lewis number (Le), Brownian motion parameter(Nb) Thermophoresis parameter (Nt), non-dimensional energy (E),temperature difference parameter (δ), dimensionless reaction rate (σ) and fitted rate constant (*n*) respectively. are discussed with the help of graphs.

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