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Cubic Graceful Labeling for Star and Bistar Related Graphs

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Abstract:

In this paper we proved some new theorems related with Cubic Graceful Labeling. A graphG(V,E) with n vertices and m edges is said to be a Cubic graceful graph if there exists an injective function$f$:V(G)→{0,1,2,3,………m^3} such that the induced mapping f:E(G):→{$1^3,2^3,3^3$,……….m^3}defined by $f(uv)=|f(u)-f(v)|$ is an injection the resulting edge labels and vertex labels are distinct. Thefunction f is called a cubic graceful labeling of G. we have proved that the star $K_{1,n}$, bistar $B_{m,n,}$ thegraph obtained by the subdivision of the edges of the star $K_{1,n}$, the graph obtained by the subdivision of thecentral edge of the bistar $B_{m,n}$ are cubic graceful graphs .

Keywords: Graceful graph, Cubic graceful labeling, Cubic graceful graph, Star graph, Bistar graph.

1. Introduction

Graph labeling have often been motivated by practical problems is one of fascinating areas of research. A systematic study of various applications of graph labeling is carried out in Bloom and Golomb [1]. . Labeled graph plays vital role to determine optimal circuit layouts for computers and for the representation of compressed data structure.

The study of graceful graphs and graceful labeling methods was introduced by Rosa[7]. Rosa defined a β-valuation of graph G with m edges as an injection from the vertices of G to the set {0,1,2,3,……..,m} such that when each edge xy is assigned the label | f(x)-f(y) |, the resulting edge labels are distinct. β- Valuations are the functions that produce graceful labeling. However, the term graceful labeling was not used until Golomb studied such labeling several years later[2].

I begin with simple, finite, connected and undirected graph G= (V, E) with n vertices and m edges. For all other standard terminology and notations I follow Harary[3].

A graph G = (V,E) with n vertices and m edges is said be ***Cubic Graceful Graph (CGG)*** if there exists an injective function $f: V(G)\rightarrow\{0,1,2,\ldots\ldots m^3\}$ such that the induced mapping $f^*: E(G)\rightarrow\{1^3,2^3,\ldots\ldots m^3\}$ defined by $f^*(uv): \rightarrow|f(u)- f(v)|$ is a bijection . The function f is called a ***Cubic Graceful Labeling (CGL)*** of G [6].

Definition 1.1

The path on n vertices is denoted by P_n.

Definition 1.2

A complete bipartite graph $K_{1,n}$ is called a star and it has (n+1) vertices and n edges

Definition 1.3

The bistar graph $B_{m,n}$ is the graph obtained from a copy of a star $K_{1,m}$ and a copy of star $K_{1,n}$ by joining the vertices of maximum degree by an edge.

II. Cubic Graceful Graphs.

Definition 2.1

A (p,q) graph G =(V,E) is said to be **Cubic Graceful Graph** if there exists an injective function f:V(G)→{0,1,2,3,………q^3} such that the induced mapping f_p : E(G)→ {1^3,2^3,3^3,……….q^3} defined by $f_p(uv)$ = | $f(u)$-$f(v)$| is an injection. The function f is called a cubic graceful labeling of G[6].

Example 2.1

The cubic graceful labeling of the P_6 graph is given in the figure 1.

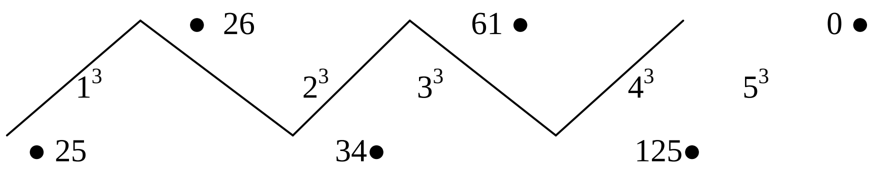

Figure 3. A cubic graceful labeling of P_6.

III. Main Results

Theorem 3.1

The star $K_{1,n}$ is cubic graceful for all n

Proof:

Let V($K_{1,n}$) = { u_i/ 1 ≤ i ≤ n+1 }

Let E($K_{1,n}$) = { u_{n+1} u_i / 1≤ i ≤ n}. Define an injection f: V($K_{1,n}$) → {0,1,2,3,………n^3} by $f(u_i)$ = i^3 if 1≤ i ≤ n and $f(u_{n+1})$ = 0. Then f induces a bijection f_p: $E(K_{1,n})$→ {1^3,2^3,3^3……….n^3}.

Example 3.1 A cubic graceful labeling of star $K_{1,8}$ is shown in Figure 2.

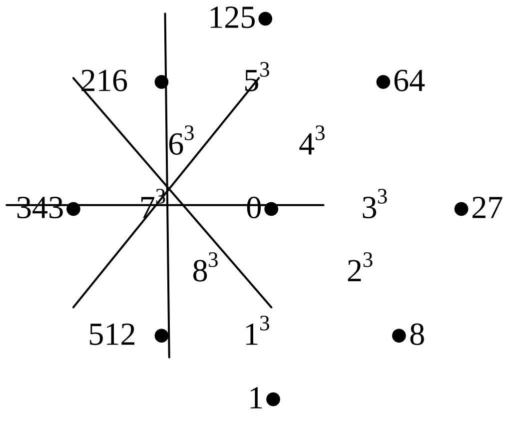

Figure 4. A cubic graceful labeling of star $K_{1,8}$.

Theorem 3.2

The graph obtained by the subdivision of the edges of the star $K_{1,n}$ is a cubic graceful graph.

Proof: Let G be the graph obtained by the subdivision of the edges of the star $K_{1,n}$. Let

$$V(G) = v, u_{i,} w_i \quad if \;\; 1 \le i \le n$$

$$E(G) = vw_i, w_{i,} u_i \quad if \;\; 1 \le i \le n$$

Define an injection $f : V(G) \to \{ 0, 1,2,3, \dots \dots \dots . .8n^3 \}$

$$f(w_i) = (2n + 1 - i)^3 \quad if \;\; 1 \le i \le n$$

$$f(u_i) = (2n + 1 - i)^3 - (n + 1 - i)^3 \quad if \;\; 1 \le i \le n$$

$f(v) = 0.$

Then, f induces a bijection $f_p : E(G) \to \{ 1^3, 2^3, 3^3, \dots \dots \dots 8n^3 \}$ and

Hence the subdivision of the edges of the star $K_{1,n}$ is a cubic graceful graph.

Example 3.2

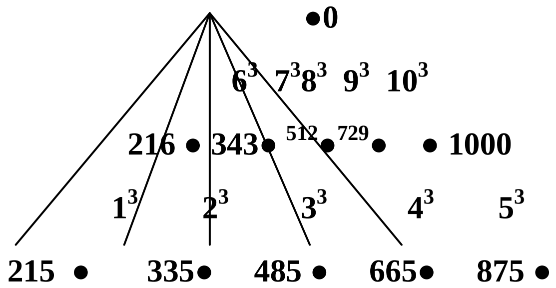

Figure 5. A cubic graceful labeling by the subdivision of edge star $K_{1,5}$.

Theorem 3.3
Every bistar $B_{m,n}$ is a cubic graceful graph.

Proof: Let $B_{m,n}$ be the bistar graph with m+n+2 vertices. Let $V(B_{m,n}) = \{u_i, v_j \,/\, 1 \le i \le m+1,\ 1 \le j \le n+1 \}$ and $E(B_{m,n}) = \{ u_i u_{m+1},\ v_j v_{n+1}, u_{m+1} v_{n+1} \,/\, 1 \le i \le m, 1 \le j \le n \}$.

Case (i): m > n .

Define an injection $f : V(B_{m,n}) \to \{0,1,2,3,\dots\dots\dots(m+n+1)^3 \}$ by

$f(u_i) = (m + n + 2 - i)^3$ if $1 \le i \le m$; $f(u_{m+1}) = 0$.

$f(v_j) = (n + 2 - j)^3 + 1$ if $1 \le j \le n$; $f(v_{n+1}) = 1$.

Following figure shows m > n pattern exhausts all possibilities and the graph under consideration admits cubic graceful labeling.

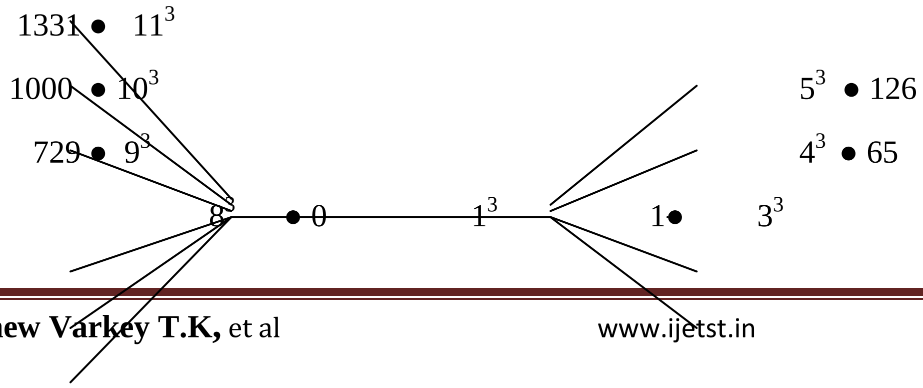

Figure 6. A cubic graceful labeling of $B_{6,4}$ (if m > n)

Case (ii) : m < n

Define an injection $f : V(B_{m,n}) \to \{0,1,2,3,\ldots\ldots\ldots(m+n+1)^3$ by

$f(u_i) = (m+2-i)^3$ if $1 \le i \le m$; $f(u_{m+1}) = 0$

$f(v_j) = (m+n+2-j)^2 + 1$ if $1 \le j \le n$; $f(v_{n+1}) = 1$.

The following figure shows m < n pattern exhausts all possibilities and the graph under consideration admits cubic graceful labeling.

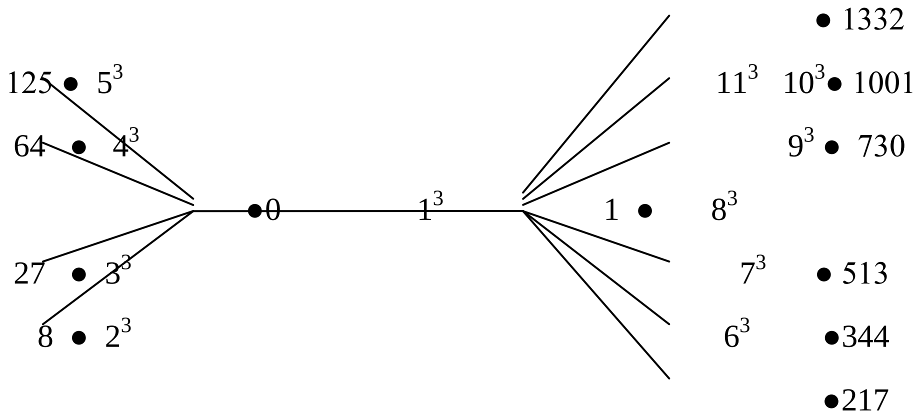

Figure 7. A cubic graceful labeling of $B_{4,6}$ (if m < n)

Case (iii): m = n.

Define an injection $f : V(B_{m,m}) \to \{0,1,2,3,\ldots\ldots\ldots(2m+1)^3\}$ by

$f(u_{m+1}) = 0$; $f(u_i) = (2m+2-i)^3$ if $1 \le i \le m$;

$f(v_{m+1}) = 1$; $f(v_j) = (m+2-j)^3 + 1$; if $1 \le j \le m$;

The following figure shows m = n pattern exhausts all possibilities and the graph under the consideration admits cubic graceful labeling .

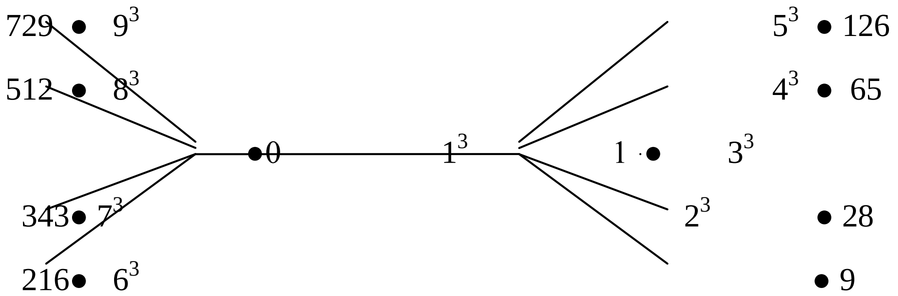

Figure 8. A cubic graceful labeling of $B_{4,4}$ (if m = n)

In all the above three cases, f induces a bijection $f_p : E(B_{m,n}) \to \{1^3, 2^3, 3^3, \ldots\ldots(m+n+1)^3\}$.

Theorem 3.4

The graph obtained by the subdivision of the central edge of the bistar $B_{m,n}$ is a cubic graceful graph.

Proof:

Let G be the graph obtained by the subdivision of the central edge of the bistar $B_{m,n}$.

Let $V(G) = \begin{cases} w, \\ u_i \quad 1 \le i \le m+1 \\ v_j, \quad 1 \le j \le n+1 \end{cases}$

Then $E(G) = \begin{cases} u_i u_{m+1}, & 1 \le i \le m \\ v_j v_{n+1}, & 1 \le j \le n \\ w u_{m+1}, \\ w v_{n+1} \end{cases}$

Case (i): m > n

Define an injection $f: V(G) \rightarrow \{ 0,1,2,3,\ldots\ldots.(m+n+2)^3 \}$ by

$f(u_{m+1}) = (m+n+2)^3$, $f(v_{n+1}) = (m+n+1)^2$, $f(w) = 0$;

$f(u_i) = (m+n+2)^3 - (m+n+1-i)^3 \quad if \quad 1 \le i \le m$

$$f(v_j) = (m+n+1)^3 - (n-j+1)^3 \quad if \quad 1 \le j \le n$$

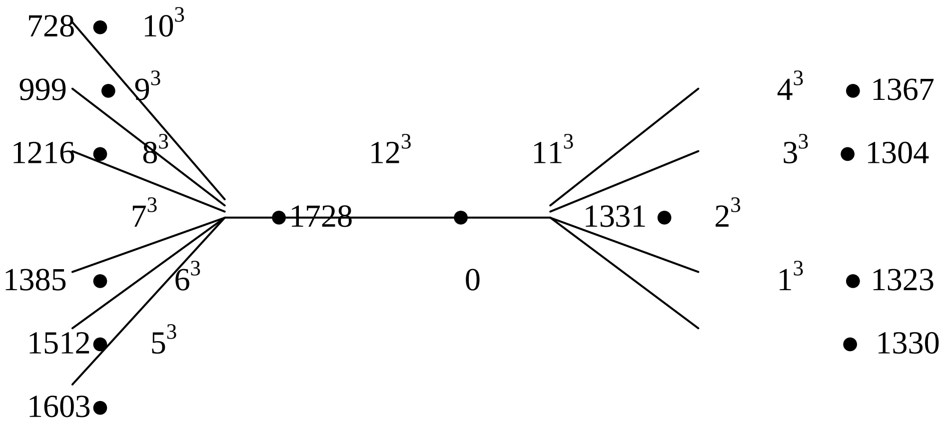

Figure 9. Acubic graceful labeling by the subdivision of central edge bistar $B_{6,4}$ (if m > n)

Case (ii): m < n

Define an injection $f: V(G) \rightarrow \{ 0,1,2,3,\ldots\ldots.(m+n+2)^3 \}$ by

$f(u_{m+1}) = (m+n+1)^3$, $f(v_{n+1}) = (m+n+2)^2$, $f(w) = 0$;

$f(u_i) = (m+n+1)^3 - (m+n+1-i)^3 \quad if \quad 1 \le i \le m$

$f(v_j) = (m+n+2)^3 - (n-j+1)^3 \quad if \quad 1 \le j \le n$

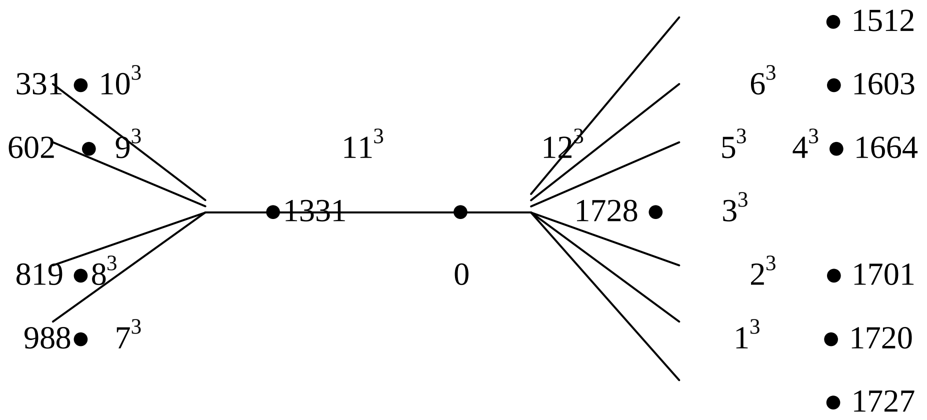

Figure 10. A cubic graceful labeling of the subdivision central edge bistar $B_{4,6}$ (if m < n)

Case (iii): m = n

Define an injection $f: V(G) \rightarrow \{ 0,1,2,3,\ldots\ldots.(2n+2)^3 \}$ by

$f(u_{m+1}) = (2n+2)^3$, $f(v_{n+1}) = (2n+1)^3$, $f(w) = 0;$

$f(u_i) = (2n+2)^3 - (2n+1-i)^3 \quad if \quad 1 \le i \le m$

$f(v_j) = (2n+1)^3 - (n-j+1)^3 \quad if \quad 1 \le j \le m$.

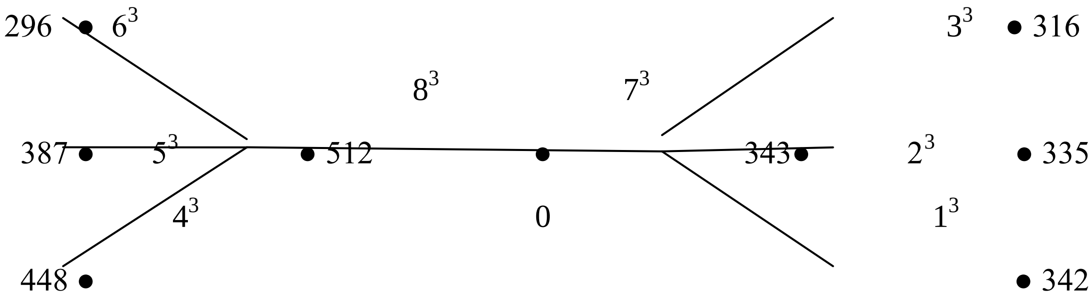

Figure 11. A cubic graceful labeling of the subdivision central edge $B_{4,4}$ (if m = n)

In all the above three cases, f induces a bijection f_p :E(G) $\rightarrow \{ 1^3,2^3,3^3,\ldots\ldots.(m+n+2)^3\}$.

IV. Conclusion

In this paper, the Cubic graceful labeling of some graphs are studied. Examples of some star graph, bistar graph observed.

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