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Cubic Harmonious Labeling Of Certain Star and Bistar Graphs

Author
Mathew Varkey T.K [1] , Mini.S.Thomas [2]
Asst.Prof, Department of Mathematics, T.K.M College of Engineering, Kollam, Kerala, India [1]
Asst. Prof , Department of Mathematics, ILM Engineering College, Eranakulam, Kerala, India [2]

Abstract
A *(n,m)* graph *G =(V,E)* is said to be **Cubic Harmonious Graph(CHG)** if there exists an injective function $f: V(G) \rightarrow \{1,2,3,\ldots\ldots m^3+1\}$ such that the induced mapping $f^*_{chg}: E(G) \rightarrow \{1^3,2^3,3^3,\ldots\ldots m^3\}$ defined by $f^*_{chg}(uv) = (f(u)+f(v)) \bmod (m^3+1)$ is a bijection. we have proved that star and bistar related graphs are cubic harmonious.
Keywords Bistar graph , Cubic harmonius labeling, Cubic harmonious graph, Star graph,.

I. Introduction

The study of graceful graphs and graceful labeling methods was introduced by Rosa[11]. Rosa defined a β-valuation of graph G with m edges as an injection from the vertices of G to the set {0,1,2,3,……..,m} such that when each edge xy is assigned the label | f(x)-f(y) |, the resulting edge labels are distinct. β- Valuations are the functions that produce graceful labeling. However, the term graceful labeling was not used until Golomb studied such labeling several years later[5]. Graham and Sloane[4] defined a *(n,m)*- graph G of order *n* and size *m* to be harmonious, if there is an injective function $f: V(G) \rightarrow Z_m$, where Z_m is the group of integers modulo *m*, such that the induced function $f^*: E(G) \rightarrow Z_q$, defined by $f^*(uv) = f(u) + f(v)$ for each edge $uv \in E(G)$ is a bijection. Square harmonious graphs were introduced in [12]. Cubic graceful graphs were introduced in [7]. Cubic harmonious graphs were defined in [8]. Throughout this paper we consider simple, finite, connected and undirected graph.

Definition 1.1
The path on n vertices is denoted by P_n.
Definition 1.2
A complete bipartite graph $K_{1,n}$ is called a star and it has (n+1) vertices and n edges
Definition 1.3
The bistar graph $B_{m,n}$ is the graph obtained from a copy of a star $K_{1,m}$ and a copy of star $K_{1,n}$ by joining the vertices of maximum degree by an edge

II. Main Results

Theorem 2.1

The graph obtained by the subdivision of the edges of stars of the bistar $B_{m,n}$ is a cubic harmonious for all m, n ≥2.

Proof :

The vertex set and the edge set of the bistar $B_{m,n}$ is defined as follows:

$V(B_{m,n}) = v_r u_s$; $1 \leq r \leq m+1$, $1 \leq s \leq n+1$

and $E(B_{m,n}) = v_r v_{m+1}, v_{m+1} u_{n+1}, u_s u_{n+1};$ $1 \leq r \leq m, 1 \leq s \leq n$

Then $| V(B_{m,n}) | = m+n+2$ and $| E(B_{m,n}) | = m+n+1$

Let G be the graph obtained by the subdivision of the edges of stars of $B_{m,n}$. Let a_r divide $v_r v_{m+1}$ for $1 \leq r \leq m$ and b_s divide $u_s u_{n+1}$ for $1 \leq s \leq n$. Then the vertex and edge set of G are given by

$V(G) = \{ v_r, u_s ;\ 1 \leq r \leq m+1, 1 \leq s \leq n+1 \}\ U\ \{ a_r, b_s ;\ 1 \leq r \leq m, 1 \leq s \leq n \}$

and

$E(G) = \{ a_r v_{m+1}, v_r a_r ;\ 1 \leq r \leq m \}\ U\ \{ v_{m+1} u_{n+1} \}\ U\ \{ b_s u_{n+1}, u_s b_s \};$ $1 \leq s \leq n$

$| V(G) | = 2m+2n+2$

$| E(G) | = 2m+2n+1$

For all cases $m<n, m=n, m>n$, we define an injection $f: V(G) \rightarrow \{1,2,\ldots\ldots.(2m+2n+1)^3+1\}$ by

$f(v_{m+1}) = (2m+2n+1)^3+1;$

$f(u_{n+1}) = (2m+2n+1)^3$

$f(a_r) = (2m+2n+1-r)^3 + (2m+2n+1)^3+1-f(v_{m+1});$ $1 \leq r \leq m$

$f(v_r) = (m+n-r+1)^3+(2m+2n+1)^3+1-f(a_r);$ $1 \leq r \leq m$

$f(b_s) = (m+2n-s+1)^3+(2m+2n+1)^3+1-f(u_{n+1});;$ $1 \leq s \leq n$

$f(u_s) = (n-s+1)^3+(2m+2n+1)^3+1-f(b_s);;$ $1 \leq s \leq n$

The induced edge mapping are

$f^*(v_{m+1} u_{n+1}) = (2m+2n+1)^3;$

$f^*(a_r v_{m+1}) = (2m+n+1-r)^3;$ $1 \leq r \leq m$

$f^*(v_r a_r) = (m+n-r+1)^3;$ $1 \leq r \leq m$

$f^*(b_s u_{n+1}) = (2m+2n-s+1)^3;$ $1 \leq s \leq n$

$f^*(u_s b_s) = (n-s+1)^3;$ $1 \leq s \leq n$

The vertex labels are in the set $\{1,2,3,\ldots\ldots.. (2m+2n+1)^3+1\}$.Then the edge labels are arranged in the set $\{1^3,2^3,3^3,\ldots\ldots\ldots\ldots. (2m+2n+1)^3\}$. So the vertex labels are and edge labels are also distinct and cubic. So the graph G is cubic harmonious for all m,n ≥ 2.

Theorem 2.2

The graph obtained by the subdivision of all edges of the bistar $B_{m,n}$ is a cubic harmonious for all m, n ≥ 2.

Proof :

The bistar $B_{m,n}$ contains $(m+n+2)$ vertices and $(m+n+1)$ edges.

So,

$V(B_{m,n}) = v_r, u_s$; $\quad 1 \le r \le m+1$, $\quad 1 \le s \le n+1$.

and $E(B_{m,n}) = v_r v_{m+1}, v_{m+1} u_{n+1}, u_s u_{n+1}$; $\quad 1 \le r \le m, 1 \le s \le n$

Let G be the graph obtained by the subdivision of all edges of the bistar $B_{m,n}$. Let a_r divide $v_r v_{m+1}$ for $1 \le r \le m$ and b_s divide $u_s u_{n+1}$ for $1 \le s \le n$. Let 'w' divide $v_{m+1} u_{n+1}$ Then the vertex and edge set of G are given by

$V(G) = \{ v_r, u_s ;\ 1 \le r \le m+1, 1 \le s \le n+1 \} \cup \{ a_r, b_s ;\ 1 \le r \le m, 1 \le s \le n \} \cup \{ w \}$

and

$E(G) = \{ a_r v_{m+1}, v_r a_r ;\ 1 \le r \le m \} \cup \{ w v_{m+1}, w u_{n+1} \} \cup \{ b_s u_{n+1}, u_s b_s ;\ 1 \le s \le n \}$

So, $|V(G)| = 2m + 2n + 3$

$|E(G)| = 2m + 2n + 2$

For all cases $m<n$, $m=n$, $m>n$, we define an injection $f: V(G) \to \{1, 2, \ldots (2m+2n+2)^3+1\}$ by

$f(v_{m+1}) = (2m+2n+2)^3+1 = 8(m+n+1)^3+1$;

$f(w) = 8(m+n+1)^3$

$f(u_{n+1}) = (2m+2n+1)^3+1$

$f(a_r) = (2m+2n+1-r)^3 + 8(m+n+1)^3+1-f(v_{m+1})$; $\quad 1 \le r \le m$

$f(v_r) = (m+n-r+1)^3+8(m+n+1)^3+1-f(a_r)$; $\quad 1 \le r \le m$

$f(b_s) = (m+2n-s+1)^3 + 8(m+n+1)^3+1-f(u_{n+1})$;; $\quad 1 \le s \le n$

$f(u_s) = (n-s+1)^3+8(m+n+1)^3+1-f(b_s)$; ; $\quad 1 \le s \le n$

The induced edge mapping are

$f^*(v_{m+1} w) = 8(m+n+1)^3$;

$f^*(w u_{n+1}) = (2m+2n+1)^3$;

$f^*(a_r v_{m+1}) = (2m+2n-r+1)^3$; $\quad 1 \le r \le m$

$f^*(v_r a_r) = (m+n-r+1)^3$; $\quad 1 \le r \le m$

$f^*(u_{n+1} b_s) = (m+2n+1-s)^3$; $\quad 1 \le s \le n$

$f^*(b_s u_s) = (n+1-s)^3$; $\quad 1 \le s \le n$

The vertex labels are in the set $\{1,2,3, \ldots\ldots (2m+2n+1)^3+1\}$.Then the edge labels are arranged in the set $\{1^3, 2^3, 3^3, \ldots\ldots (2m+2n+1)^3\}$. So the vertex labels are and edge labels are also distinct and cubic. So the graph G is cubic harmonious for all m,n $\ge$ 2.

Theorem 2.3

The graph obtained by the subdivision of the edges of the path P_n in comb $P_n \odot K_1$ is cubic harmonious for all $n \ge 2$

Proof:

Let G be the graph obtained by the subdivision of the edges of the path P_n in comb $P_n \odot K_1$.

Let $V(G) = \begin{cases} v_r, u_r & 1 \leq r \leq n, \\ z_s\ ; & 1 \leq s \leq n\text{-}1 \end{cases}$

and

$E(G) = \begin{cases} v_r z_r, z_r v_{r+1}\ ; & 1 \leq r \leq n\text{-}1 \\ v_s u_s\ ; & 1 \leq s \leq n \end{cases}$

So $|V(G)| = 3n\text{-}1$ and $|E(G)| = 3n\text{-}2$

Define an injection $f : V(G) \rightarrow \{1,2,3,\ldots\ldots,(3n\text{-}2)^3+1\}$ by

$f(v_1) = (3n\text{-}2)^3+1$

$f(z_1) = (3n\text{-}2)^3\ ;$

$f(v_{r+1}) = (3n\text{-}2r\text{-}1)^3 + (3n\text{-}2)^3+1 - z_r\ ;$ $1 \leq r \leq n-1$

$f(z_s) = (3n\text{-}2s)^3 + (3n\text{-}2)^3+1 - f(v_s)\ ;$ $1 \leq s \leq n-1$

$f(u_r) = r^3 + (3n\text{-}2)^3+1 - f(v_s)\ ;$ $1 \leq r \leq n$

Then f induces a bijection $f^*: E(G) \rightarrow \{1^3,2^3,3^3\ldots\ldots\ldots(3n\text{-}2)^3\}$.

The edge labels are as follows.

$f^*(v_r z_r) = (3n\text{-}2r)^3;$ $1 \leq \leq n-1$

$f^*(z_r v_{r+1}) = (3n\text{-}2r\text{-}1)^3;$ $1 \leq r \leq n\text{-}1$

$f^*(v_r u_r) = (r)^3;$ $1 \leq r \leq n$

The edge labels are distinct and cubic, they are $\{1^3,2^3,3^3\ldots\ldots\ldots(3n\text{-}2)^3\}$.
Hence the theorem.

Theorem 2.4

The graph $<S_m{:}n>$ is a cubic harmonious for all $m, n \geq 1$.

Proof:

Let $\{u, u_{01,\ldots\ldots\ldots,}u_{0r,}.\ u_{11},u_{12,\ldots\ldots\ldots}u_{1s},\ u_{21},u_{22,\ldots\ldots\ldots\ldots,}u_{2s,\ldots\ldots\ldots}\ u_{rs}\}$ be the vertices of the r^{th} copy of the star S_m in $<S_m{:}\ n>$ where u_{or} is the centre of vertex of the star for $1 \leq r \leq n$, $1 \leq s \leq m$

Let $V(G) = u, u_{or}, u_{rs};$ $1 \leq r \leq n;\ 1 \leq s \leq m$

and $E(G) = \begin{cases} uu_{or}; & 1 \leq r \leq n; \\ u_{or}u_{rs}; & 1 \leq r \leq n;\ 1 \leq s \leq m \end{cases}$

So $|V(G)| = n(m+1)+1$ and $|E(G)| = n((m+1)$

Define an injection $f : V(< s_m : n>) \rightarrow \{1,2,\ldots\ldots, n(m+1)^3+1\}$ by

$$f(u) = n(m+1)^3 + 1$$

$$f(u_{0r}) = [n(m+1)-r+1]^3; \qquad 1 \le r \le n$$

$$f(u_{rs}) = [m(n-r+1)-(s-1)]^3 + n(m+1)^3 + 1 - f(u_{or}); \qquad 1 \le r \le n,\ 1 \le s \le m \ ;$$

Then f induces a bijection $f^*: E(<s_m : n) \rightarrow \{1^3, 2^3 \ldots\ldots [n(m+1)]^3 + 1\}$

The induced edge labels of the given graph G are as follows:

$$f^*(uu_{or}) = [n(m+1)-r+1]^3; \qquad 1 \le r \le n$$

$$f^*(u_{or}\, u_{rs}) = \left[m\left(n-r+1)-(s-1)\right)\right]^3; \qquad 1 \le r \le n,\ 1 \le s \le m$$

The edge labels are distinct and cubic. Hence the graph $<s_m:n>$ is a cubic harmonious

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