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A Proposed Probability Model for the Single Male out Migrants from Rural to Urban Areas

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Abstract
The objective of the current work to develop a probability model under the certain assumptions. The model involves several parameters and these are estimated by different method of estimation techniques. Application is made to set of demographic data.

Introduction

One important facet of study on population is the study of migration arising out of various social, economic or political reasons. For a large country like India, the study of movement of population in different parts of the country helps in understanding the dynamics of the society better. At this junction in the economic development, in the country, especially when many states are undergoing faster economic development, particularly in areas, such as, manufacturing, information technology or service sectors, data migration profile of population has become more important.

A result, the micro level analysis of migration data have several implications for regional planning, housing policies and sociological models (Pryor, 1975), and have played a decisive role to the development of the theory of migration (Dejong and Dardner, 1981).Recently the study of human grouping has become an important subject for research which provides an understanding of the social, economic and cultural and demographic chrematistics of the people. Perhaps the most important human grouping that may be of concern in the population studies, In general and migration decision process in particularly, is a household. A house hold has been defined, by several developing countries, a basic socio-economic unit for the integrated rural development and consequently data are being collected at this level (Robbani, 1979). Naturlly, the numbers of migrants who have great attachment to their homeland have important bearing on the economic and social development of the household. For example, a household which at list one migrant is more susceptible to have new ideas and cultural contacts than a household having no migrant (Singh, 1986).

The micro level studies of the out migration have been undertaken at a community level, village level, household level depending upon the objective and availability of data. In this direction Singh and Yadava, 1981, Sharma, 1987, Ojha and Pandey, 1991, Pandey 2004 has been proposed model for the rural out migration.

The main objective of the present work is to develop inflated probability model for the single male migrants' age 15 years and above from households takinginto account the risk of migration due to their establishment in the rural areas.

Probability Model

1) To develop a inflated probability model for a single male migrants aged 15 years and above from the rural areas to urban areas under the following assumptions.
2) Let α be the probability that a household is exposed to the risk of migration at the survey point 1- α be the probability that a household is not expose to risk of migration.
3) Out of α proportion of households, Let β proportion of household in which only one migrate.

The number of single male migrants from households follows displaced Poisson distribution as:

$$P(X=k)=\alpha(1-\beta)e^{-\theta}.\frac{\theta^{k-2}}{(k-2)!} \quad \ldots\ldots(2.1)$$

k=2, 3, 4.........

Under these assumptions (1) to (3), the probability model for the number of single male migrant X, is given by

$$\left.\begin{aligned} P(X=0)&=(1-\alpha)\\ P(X=1)&=\alpha.\beta\\ P(X=k)&=\alpha(1-\beta)e^{-\theta}.\frac{\theta^{k-2}}{(k-2)!}\\ &\quad k=1,2,3\ldots\ldots \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ (2.2)$$

whereα, β and θ parameters of the proposed model (2.2)

3. Estimation

The proposed model (2.2) consists of three parameter α, β and θ. These parameters are estimated by method of moments and method of maximum likelihood estimation, which is explained under the following:

(a) Method of Moment

The proposed probability model consist three parameters α, β and θ .These parameter are estimated by equating$Zero^{th}$, first cell theoretical frequencies to the observed frequencies and observed mean with their corresponding theoretical value. Which convert in to following equations:

$$\frac{f_0}{f}=1-\alpha \ldots\ldots\ldots(3.1)$$

$$\frac{f_1}{f}=\alpha\beta \ldots\ldots\ldots(3.2)$$

$$\text{Mean }(\bar{X})=\alpha\beta+(1-\beta)(\theta+2) \ldots\ldots\ldots(3.3)$$

Where

f_0= Number of obseration in Zeroth cell.

f_1=Number of observation in first cell

f=Total Number observation.

$\bar{X}$=Observed mean

(b) Method of Maximum Likelihood Estimator:

The likelihood function of the probability model can be expressed as:

$$L = (1-\alpha)^{f_0}(\alpha.\beta)^{f_1}[(1-\beta)\alpha.\beta]^{f_2}[1-\{(1-\alpha)+\alpha.\beta+(1-\beta)\alpha.e^{-\theta}]^{(f-f_0-f_1-f_2)} \qquad \ldots\ldots\ldots.(3.4)$$

Taking log both sides, we get

$$\ln L = f_0 \ln 1-\alpha) + f_1 \ln\alpha + f_1 \ln\beta + f_2 \ln(1-\beta) + f_2 \ln\alpha - \theta f_2 + (f - f_0 - f_1 - f_2)\ln(1-\beta) + (f - f_0 - f_1 - f_2)\ln\alpha + (f - f_0 - f_1 - f_2)\ln(1-e^{-\theta}) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(3.5)$$

Partially differentiating with respect to α, β and θ and equating to zero. This results with following equations:

$$\frac{\partial}{\partial\alpha}\ln L = f_0\left[\frac{-1}{1-\alpha}\right] + \frac{f_1}{\alpha} + \frac{f_2}{\alpha} + \frac{(f - f_0 - f_1 - f_2)}{\alpha} = 0 \qquad \ldots..(3.6)$$

This gives

$$\hat{\alpha} = \frac{(f - f_0)}{f} \qquad \ldots\ldots\ldots\ldots\ldots\ldots..(3.7)$$

$$\frac{\partial}{\partial\beta}\ln L = \frac{f_1}{\beta} + f_2\left[\frac{-1}{1-\beta}\right] + (f - f_0 - f_1 - f_2)\left[\frac{-1}{1-\beta}\right] = 0 \qquad \ldots..(3.8)$$

This gives

$$\hat{\beta} = \frac{f_1}{(f - f_0)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(3.9)$$

$$\frac{\partial}{\partial\theta}\ln L = -f_2 + (f - f_0 - f_1 - f_2).\frac{1}{(1-e^{-\theta})}e^{-\theta} = 0 \qquad \ldots\ldots..(3.10)$$

This gives

$$\hat{\theta} = (-1)\left[\ln\frac{f_2}{(f - f_0 - f_1)}\right] \qquad \ldots\ldots\ldots.(3.11)$$

$\hat{\alpha}, \hat{\beta}, \hat{\theta}$ are maximum likelihood estimators of α β and θ respectively

Now

$$\frac{\partial^2}{\partial\alpha^2}\ln L = -\left[\frac{f_0}{(1-\alpha)^2}+\frac{(f-f_0)}{\alpha^2}\right]$$

$$\frac{\partial^2}{\partial\beta^2}\ln L = -\left[\frac{f_1}{\beta^2}+\frac{(f-f_0-f_1}{(1-\beta)^2}\right] \frac{\partial^2}{\partial\theta^2}\ln L = -\left[\frac{e^{-\theta}(f-f_0-f_1-f_2)}{(1-e^{-\theta})^2}\right] \quad \ldots(3.12)$$

Now

$$-\frac{E\left(-\frac{\partial^2}{\partial\alpha^2}\ln L\right)}{f} = \left[\frac{1}{1-\alpha}+\frac{1}{\alpha}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.13)$$

$$-\frac{E\left(-\frac{\partial^2}{\partial\beta^2}\ln L\right)}{f} = \left[\frac{\alpha}{\beta(1-\beta)}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.14)$$

$$-\frac{E\left(-\frac{\partial^2}{\partial\theta^2}\ln L\right)}{f} = \left[\frac{\alpha e^{-\theta}(1-\beta)(1-\theta)}{(1-e^{-\theta})^2}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots(3.15)$$

Ultimately we get,

$$E(f_0) = f(1-\alpha)$$

$$E(f_1) = f\alpha\beta$$

$$E(f_2) = f(1-\beta)\alpha e^{-\theta} (3.16) E(f-f_0-f_1-f_2) = f\left[\alpha\{(1-\beta)(1-e^{-\theta})\}\right]$$

After estimating the all parameters involved in the proposed model by MLE, then we are interested to calculate the variances of each estimated parameters. For this purpose we estimate the variances of two parameter supposing one is known. There are three cases arise as following:

Case1: When α is known, The Fisher Information Matrix (FIM) is given by:

FIM (β,θ) = ……………………(

$$\begin{bmatrix} \dfrac{-E\left[\dfrac{\partial^2}{\partial\beta^2}\ln L\right]}{f} & 0 \\ 0 & \dfrac{-E\left[\dfrac{\partial^2}{\partial\theta^2}\ln L\right]}{f} \end{bmatrix}$$

$$= \begin{bmatrix} \left[\dfrac{\alpha}{\beta(1-\beta)}\right] & 0 \\ 0 & \left[\dfrac{\alpha e^{-\theta}(1-\beta)(1-\theta)}{(1-e^{-\theta})^2}\right] \end{bmatrix} = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix}$$

$$\phi_{12} = \left[\frac{\partial^2}{\partial\beta\partial\theta}\ln L\right] = 0$$

$$\phi_{21} = \left[\frac{\partial^2}{\partial\theta\partial\beta}\ln L\right] = 0$$

$$V(\hat{\theta}) = \frac{1}{f}\left[\frac{\phi_{11}}{\phi_{11}.\phi_{22}}\right] \quad \text{……………………..(3.17)}$$

$$V(\hat{\beta}) = \frac{1}{f}\left[\frac{\phi_{22}}{\phi_{11}.\phi_{22}}\right] \quad \text{................................(3.18)}$$

Case 2: when β is known, Fisher Information Matrix (FIM) is given by

$$\text{FIM }(\alpha,\theta)= \begin{bmatrix} \dfrac{-E\left[\dfrac{\partial^2}{\partial\alpha^2}\ln L\right]}{f} & 0 \\ 0 & \dfrac{-E\left[\dfrac{\partial^2}{\partial\theta^2}\ln L\right]}{f} \end{bmatrix}$$

$$= \begin{bmatrix} \left[\dfrac{1}{1-\alpha}+\dfrac{1}{\alpha}\right] & 0 \\ 0 & \left[\dfrac{\alpha e^{-\theta}(1-\beta)(1-\theta)}{(1-e^{-\theta})^2}\right] \end{bmatrix} = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix}$$

$$\phi_{12} = \left[\frac{\partial^2}{\partial\alpha\partial\theta} \ln L \right] = 0$$

$$\phi_{21} = \left[\frac{\partial^2}{\partial\theta\partial\alpha} \ln L \right] = 0$$

$$V(\hat{\theta}) = \frac{1}{f}\left[\frac{\phi_{11}}{\phi_{11}.\phi_{22}} \right] \quad \text{.......................(3.19)}$$

$$V(\hat{\alpha}) = \frac{1}{f}\left[\frac{\phi_{22}}{\phi_{11}.\phi_{22}} \right] \quad \text{.......................(3.20)}$$

Case 3: when θ is known, Fisher Information Matrix (FIM) is given by

$$\text{FIM }(\alpha,\beta)= \begin{bmatrix} \frac{-E\left[\frac{\partial^2}{\partial\alpha^2}\ln L\right]}{f} & 0 \\ 0 & \frac{-E\left[\frac{\partial^2}{\partial\beta^2}\ln L\right]}{f} \end{bmatrix}$$

$$= \begin{bmatrix} \left[\frac{1}{1-\alpha} + \frac{1}{\alpha}\right] & 0 \\ 0 & \left[\frac{\alpha}{\beta(1-\beta)}\right] \end{bmatrix} = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix}$$

$$\phi_{12} = \left[\frac{\partial^2}{\partial\alpha\partial\beta} \ln L \right] = 0$$

$$\phi_{21} = \left[\frac{\partial^2}{\partial\beta\partial\alpha} \ln L \right] = 0$$

$$V(\hat{\beta}) = \frac{1}{f}\left[\frac{\phi_{11}}{\phi_{11}.\phi_{22}} \right] \quad \text{..…..…………………….(3.21)}$$

$$V(\hat{\alpha}) = \frac{1}{f}\left[\frac{\phi_{22}}{\phi_{11}.\phi_{22}} \right] \quad \text{……………………….(3.22)}$$

Table 1: Observe and expected distribution of the numbers of households according to single male migrants from a household in semi-urban type villages.

Number of male migrants aged 15 and over	Semi-urban Type of village		
	Observed	Expected	
		Method of Moment	Method of Maximum Likelihood
0	1042	1042	1042
1	95		
2	19	95	95
3	10		
4	2	16.44	19
5	2		
6	0 } 5	11.97	11.05
7	1		
8	0		
		5.65	3.94
Total	1171	1171.0600	1171.00
$\hat{\alpha}$		0.1102	0.1101
$\hat{\beta}$		0.7359	0.7364
$\hat{\theta}$		0.7286	0.5819
χ^2		1.1433	0.3840
d.f		1	1

Table II: observe and expected distribution of the numbers of households according to single male migrants from a household in Remote Type of village

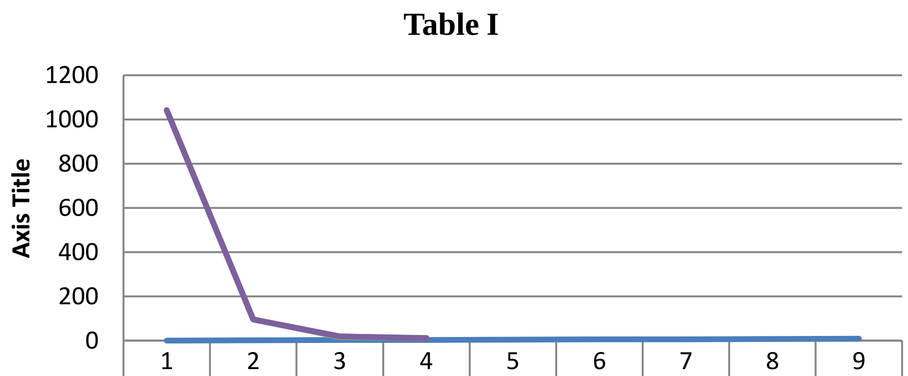
Number of child dead	Remote Type of village		
	Observed number of family	Expected	
		Method of Moment (Expected no. of families)	Method of Maximum Likelihood (Expected no. of families)
0	872	872	872
1	176		
2	59	176	176
3	18		
4	6 } 10	53.57	59
5	4 }		
6	0 }	25.96	22.91
7	0 }		
8	0 }		
		7.53	5.08
Total	1135	1135.06	1135
$\hat{\alpha}$	0.23170.2317		
$\hat{\beta}$	0.6690.6692		
$\hat{\theta}$	0.48520.3883		
χ^2	3.7995.8012		
d.f		1	1

Table III: observe and expected distribution of the numbers of households according to single male migrants from a household Growth Centre Type of village.

Number of child dead	Growth Centre Type of village		
	Observed number of family	Expected	
		Method of Moment (Expected no. of families)	Method of Maximum Likelihood (Expected no. of families)
0	978	978	978
1	154		
2	47	154	154
3	18	42.53	47
4	9 }		
5	1 }	23.8	22.58
6	0 } 11		
7	1 }	9.67	6.41
8	0 }		
Total	1208	1208	1208
$\hat{\alpha}$		0.1904	0.1903
$\hat{\beta}$		0.6696	0.6695
$\hat{\theta}$		0.5802	0.4805
χ^2		2.0667	4.21368121
d.f		1	1

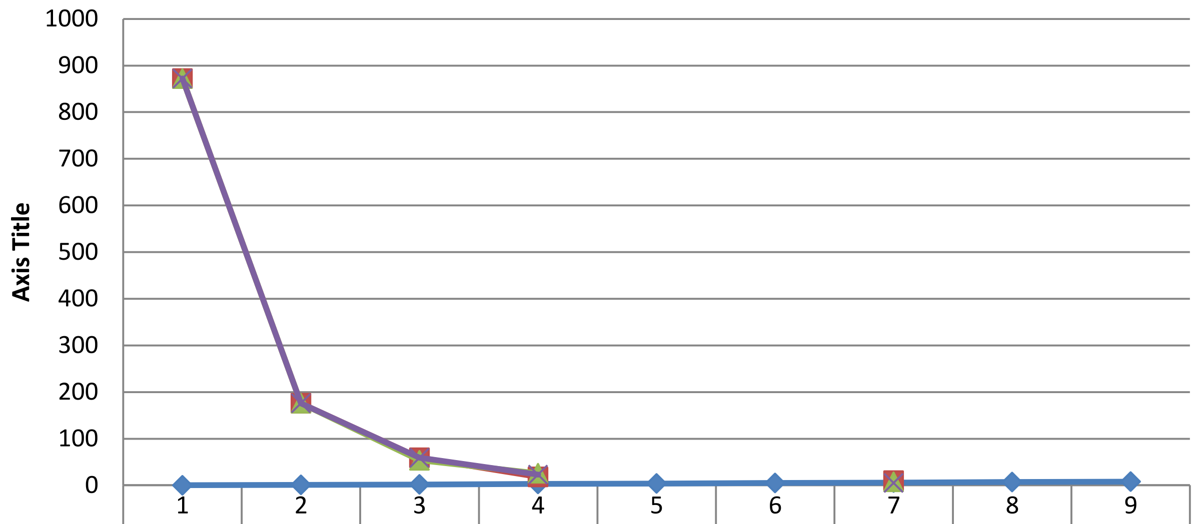
Table IV Variances of different parameters in case of different villages.

Case1: When α is known, The Fisher Information matrix is given by		
Table I	Table II	Table III
$V(\hat{\beta})$=0.021659827 $V(\hat{\theta})$=0.002782122	$V(\hat{\beta})$==0.007691972 $V(\hat{\theta})$=0.002870366	$V(\hat{\beta})$=0.000961946 $V(\hat{\theta})$=0.005964267
Case2: when β is known, The Fisher Information matrix is given by		
TableI	Table II	Table III
$V(\hat{\alpha})$=8.37118E-05 $V(\hat{\theta})$=0.024502664	$V(\hat{\alpha})$=0.00015685 $V(\hat{\theta})$=0.002870366	$V(\hat{\alpha})$=0.000127604 $V(\hat{\theta})$=0.005964267
Case3: when θ is known, The Fisher Information Matrix(FIM) is given by		
Table I	Table II	Table III
$V(\hat{\beta})$=0.021659827 $V(\hat{\alpha})$=8.37118E-05	$V(\hat{\beta})$=0.007691972 $V(\hat{\alpha})$=0.00015685	$V(\hat{\beta})$=0.000961946 $V(\hat{\alpha})$=0.000127604



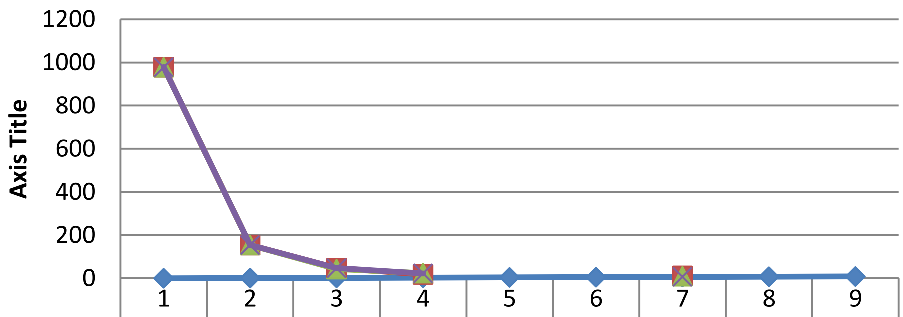
	1	2	3	4	5	6	7	8	9
X	0	1	2	3	4	5	6	7	8
F	1042	95	19	10			5		
M	1042	95	16.44	11.97			5.65		
L	1042	95	19	11.057			3.9435		

Table II



X	0	1	2	3	4	5	6	7	8
F	872	176	59	18			10		
M	872	176	53.57	25.96			7.53		
L	872	176	59	22.91387			5.08613		

Table III



X	0	1	2	3	4	5	6	7	8
F	978	154	47	18			11		
M	978	154	42.53	23.8			9.67		
L	978	154	47	22.588			6.4125		

Conclusion

Table I,II and III shows the distribution of the observed and estimated frequencies the semi-urban, remote and growth centre type villages respectively. The estimated values of the parameter α, β and θ for three types of villages are given in respective table. It is observe that the risk of migration, α, is found for remote is greater than the growth centre and semi-urban types of villages. From table I,II and III, it is found that the observed value of chi-square are insignificant at 5% and 2% level of significance and hence indicating the suitability of the proposed model on these three types of villages. Table IV shows the elements of the Fisher Information Matrix for the three types of villages. The graphs shows the deviation of the expected values from the observed values in the three types of villages in which parameters are estimated by the method of moment and MLE.The proposed models become useful tool for the prediction of future population of the urban areas due to migration, which leads to fresh environment and better health condition of the urban society.

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