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S – Valued Semi Homomorphism

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ABSTRACT

In our earlier papers [4] and [3], we have introduced the notion of Semiring –valued graphs and homomorphisms on S-valued graphs. In this paper, we discuss the notion of S-vertex homomorphism and S-edge homomorphisms on S-valued graphs and prove some simple results.

Keywords: *S-vertex homomorphism, S-edge homomorphism, S-valued semihomomorphism.*

AMS Classification: 05C25, I6Y60.

1.INTROUDUCTION

Combinatorial counting problems on graphs are important in their own right and for the application to the statistical physics. In particular, counting proper graph colourings is a classical problem and is closely associated with the problem of evaluating the particular function of the Pott's Model in statistical physics [5]. Many such counting problem can be restated as counting the number of homomorphisms from the graph of interest G to a particular fixed graph H. In our earlier paper, we discussed the notion of S-valued homomorphism between two S-valued graphs where we persumed that there is an isomorphism between two crisp graphs and we considered the semiring homomorphism between the two semirings S_1 andS_2. In this paper, we restrict the map between the vertex sets of the crisp graphs to be a homomorphism but not an isomorphism.

2. PRELIMINARIES

In this section, we recall some basic definitions that are needed for our work.

Definition 2.1.[1] Let (s,+,.) be an algebraic system with a non-empty set S together with two binary operators '+' and '.' such that

(1) (S,+,0) is a monoid

(2) (S, .) is a semigroup

(3) For all a,b,c$\in$ S, a$\cdot$(b+c) = a$\cdot$b+a$\cdot$c and (a+b) $\cdot$c = a.c+b.c.

(4). 0$\cdot$x = x.0=0$\forall$x$\in$S.

Definition 2.2 let (S, +, .) be a semiring. $\preccurlyeq$ is said to be a canonical preorder if for a, b $\in$ S, a $\preccurlyeq$ b if and only if there exists c $\in$ S such that a + c = b.

Definition 2.3.[4] Let G = (V,E $\subset V \times V$) be the underlying graph with V, E $\neq\emptyset$. For any semiring (S, +, $\cdot$), a Semiring –valued graph (or a S-valued graph) G^S is defined to be the graph G^S = (V, E, σ, ψ) where σ : V → S and ψ : E → S is defined to be

$$\psi(x,y) = \begin{cases} \min\{\sigma(x),\sigma(y)\} & if\ \sigma(x) \preccurlyeq \sigma(y) or\ \sigma(y) \preccurlyeq \sigma(x) \\ 0 & otherwise \end{cases}$$

for every unordered pair (x, y) of E $\subset V \times V$. We call σ, a S- vertex set and ψ, a S- edge set of S-valued graph G^S.

Definition 2.4.[1] Let $G_1 = (V_1,E_1)$ and $G_2 = (V_2,E_2)$ be given two graphs. A mapping

$\alpha : V_1 \rightarrow V_2$ is said to be a graph homomorphism if $(u_1, v_1) \in E_1 \Rightarrow (\alpha(u_1), \alpha(v_1)) \in E_2$.

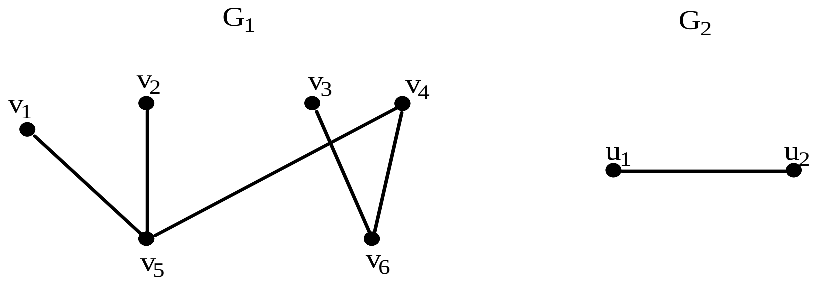
Remark 2.5. A graph homomorphism is an edge preserving map. It need not be 1-1, onto or both.

Example 2.6 Let $G_1 = (V_1, E_1)$ where $V_1 = (v_1,v_2,v_3,v_4,v_5,v_6)$ and $E_1 = \{ (v_1,v_5), (v_2,v_5), (v_3, v_6), (v_4,v_5), (v_4, v_6)\}$.

Therefore $O(V_1) = 6$ and $O(E_1) = 5$.

Let $G_2 = (V_2, E_2)$ where $V_2 = \{ u_1, u_2\}$ and $E_2 = \{(u_1, u_2)\}$

Therefore $O(V_2) = 2$ and $O(E_2) = 1$.



Define $\alpha : V_1 \rightarrow V_2$ by $v_1 \rightarrow u_1; v_2 \rightarrow u_1; v_3 \rightarrow u_1; v_4 \rightarrow u_1; v_5 \rightarrow u_2; u_6 \rightarrow u_2$ then all edges in G_1 mapped into $(u_1, u_2) \in E_2$.

Therefore for all $(v_i, v_j) \in E_1 \Rightarrow (\alpha(v_i), \alpha(v_j)) \in E_2$.

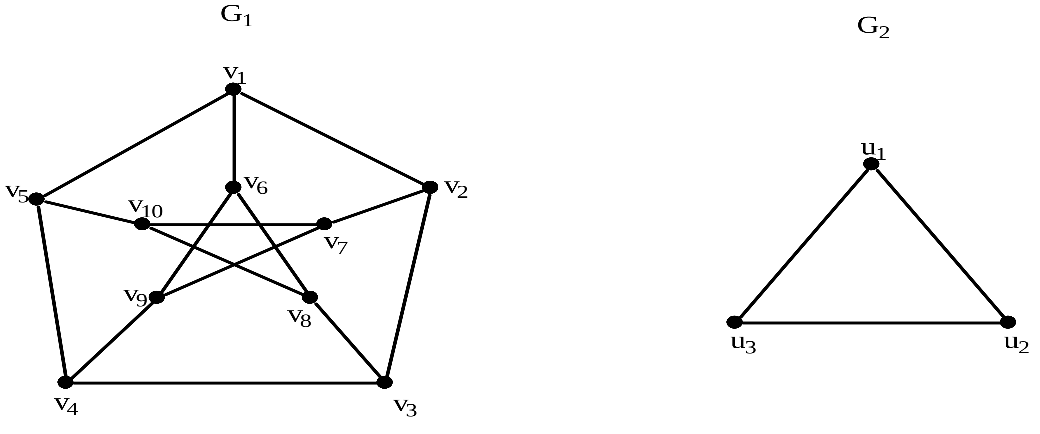
$\Rightarrow \alpha$ is a graph homomorphism and is not 1-1.

Example 2.7. Consider the Petersen graph $G_1 = (V_1\ E_1)$ where $V_1 = \{v_1, v_2, \ldots., v_{10}\}$ and $E_1 = \{(v_1, v_5), (v_1, v_6), (v_1,v_2), (v_2,v_7), (v_2,v_3), (v_3,v_8), (v_3,v_4), (v_4,v_9), (v_4,v_5), (v_5,v_{10}), (v_6,v_9), (v_6,v_8) (v_7, v_{10}), (v_7, v_9), (v_8, v_{10})\}$.

Therefore $O(V_1) = 10$: $O(E_1) = 15$.

Let $G_2 = K_3 = (V_2, E_2)$ where $V_2 = \{u_1, u_2\ u_3\}$ and $E_2 = \{(u_1, u_3), (u_1, u_2), (u_2, u_3)\}$

Therefore $O(V_2) = 3$, $O(E_2) = 3$.



Define $\alpha : V_1 \rightarrow V_2$ by $v_1 \mapsto u_1; v_2 \mapsto u_2; v_3 \mapsto u_1; v_4 \mapsto u_2; v_5 \mapsto u_3; v_6 \mapsto u_2;$

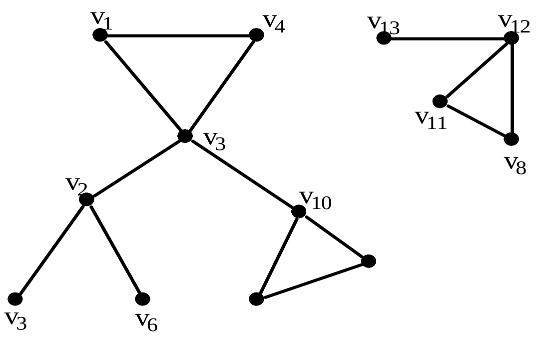
$v_7 \mapsto u_1; v_8 \mapsto u_3; v_9 \mapsto u_3; v_{10} \mapsto u_2.$

Clearly , for every $(v_i, v_j) \in E_1 \Rightarrow (\alpha(v_i), \alpha(v_j)) \in E_2$.

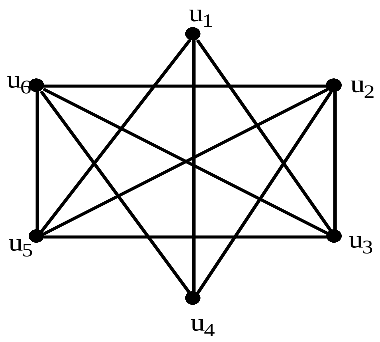
Therefore α is a homomorphism but not bijective.

Example 2.8. Let G_1 and G_2 be as follows.

G_1



G_2



Define $\alpha : V_1 \rightarrow V_2$ by

$v_1 \mapsto u_1$; $v_2 \mapsto u_2$; $v_3 \mapsto u_3$; $v_4 \mapsto u_5$; $v_5 \mapsto u_6$; $v_6 \mapsto u_5$; $v_7 \mapsto u_2$; $v_8 \mapsto u_1$; $v_9 \mapsto u_3$; $v_{10} \mapsto u_6$; $v_{11} \mapsto u_5$; $v_{12} \mapsto u_3$; $v_{13} \mapsto u_6$.

Clearly, $(v_i\, v_j) \in E_1 \Rightarrow (\alpha(v_i), \alpha(v_j)) \in E_2$

$\Rightarrow \alpha$ is a graph homomorphism but not bijective

This homomorphism $\alpha : G_1 \rightarrow G_2 \subset K_6$ is an inclusion in k_6

Definition 2.9. Let $(S_1, +, \cdot)$ and $(S_2, +, \cdot)$ be given two semirings. A mapping $\beta : S_1 \rightarrow S_2$ is a semiring homomorphism if $\beta(0_{S_1}) = 0_{S_2}$; $\beta(a + b) = \beta(a) + \beta(b)$; $\beta(a.b) = \beta(a) . B(b)$ $\forall$ a, b $\in S_1$.

Remark 2.10. If the semirings contains multiplicative identity then $\beta(1_{S_1}) = 1_{S_1}$ must be satisfied.

3. S-VALUED SEMI HOMOMORPHISM

In this section, we introduce the notion of S – valued vertex homomorphisms and S-valued edge homomorphism and S – valued semi homomorphisms on S-valued graphs and prove some simple results.

Definition3.1. Let $G_1^{S_1} = (V_1, E_1, \sigma_1, \psi_2)$ and $G_2^{S_2} = (V_2, E_2, \sigma_2, \psi_2)$ be given two s – valued graphs. A maping $\phi = (\alpha, \beta) : G_1^{S_1} \rightarrow G_2^{S_2}$ is a S – **valued vertex homomorphism if**

(i) $\alpha : V_1 \rightarrow V_2$ **is a graph homomorphism.**

(ii) $\beta : S_1 \rightarrow S_2$ **is a semiring homomorphism with** $\beta(\alpha_1(v)) = \sigma_2(\alpha(v)) = \forall\, v \in V_1$

Example 3.2 Let $S_1 = (\{0,a,b\}, +, \cdot)$ be a semiring with binary operators '+' and '.' given in the following Cayley's Tables:

+	0	a	b
0	0	a	b
a	a	a	b
b	b	b	b

.	0	a	b
0	0	0	0
a	0	a	b
b	b	b	b

Here $0 \preccurlyeq 0$, $0 \preccurlyeq a$, $0 \preccurlyeq b$, $a \preccurlyeq a$, $a \preccurlyeq b$, $b \preccurlyeq b$.

Clearly S_1 is both multiplicatively and additively commutative and hence it is a commutative semiring. Further here 0 is the additive identity element and a is the multiplicative identity element. And a and b are both additively and multiplicatively idempotent elements.

Let $S_2 = (\{0, f, g, h\}, +,\cdot)$ be a semiring whose binary operators '+' and '.' are defined as follows.

+	0	f	g	h
0	0	f	g	h
f	f	f	h	h
g	g	h	h	h
h	h	h	h	h

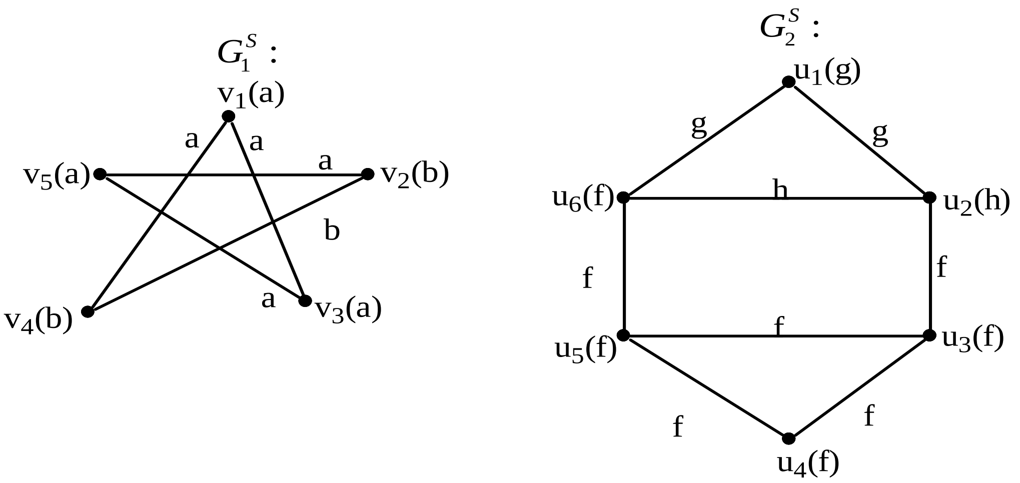
.	0	f	g	h
0	0	0	0	0
f	0	f	g	h
g	0	g	h	h
h	0	h	h	h

Here $0 \preccurlyeq f, g, h$; $f \preccurlyeq f, h$; $g \preccurlyeq g, h$; $h \preccurlyeq h$.

Clearly f is the unit element in S_2

Define $\beta : S_1 \longmapsto S_2$ by $0_1 \longmapsto 0_2$; $a \longmapsto f : b \longmapsto h$.

Then β is a semiring homomorphism which is one – one but not onto.



Define $\alpha : V_1 \rightarrow V_2$ by $v_1 \longmapsto u_5$; $v_2 \longmapsto u_2$; $v_3 \longmapsto u_4$; $v_4 \longmapsto u_6$; $v_5 \longmapsto u_3$

Then $(v_{i,} v_j) \in E_1 \Rightarrow (\alpha(v_i), \alpha(v_j)) \in E_2$

Therefore α is a graph homomorphism

Now, $\beta(\sigma_1(v_1) = \beta(a) = f = \sigma_2(\alpha(v_1))$

$\beta(\sigma_1(v_2)) = \beta(b) = h = \sigma_2(\alpha(v_2))$

$\beta(\sigma_1(v_3)) = \beta(a) = f = \sigma_2(\alpha(v_3))$

$\beta(\sigma_1(v_4)) = \beta(b) = h = \sigma_2(\alpha(v_4))$

$\beta(\sigma_1(v_5)) = \beta(a) = f = \sigma_2(\alpha(v_5))$

$\Rightarrow \beta$ is a semiring homomorphism with $\beta(\sigma_1(v_i) = \sigma_2(\alpha(v_i))\ \forall\ v_i \in E_1$

Therefore $\phi = (\alpha, \beta)$ is a S – valued vertex homomorphism.

Remark 3.3. In the above example, the vertex u_1 in V_2 is not an image of any vertex in G_1 and hence it may assume any value as weight from S_2.

Therefore even if $G_1^{S_1}$ is a S-vertex regular S-vertex homomorphism need not be, in general, preserve S-vertex regularity and hence S-regularity.

Definition 3.4. Let $G_1^{S_1}$ = (V_1 E_1 σ_1, ψ_1) and $G_2^{S_2}$ = (V_2 $E_2\sigma_2$, ψ_2) be given two S-valued graphs. A mapping $\phi = (\alpha,\beta) : G_1^{S_1} \to G_2^{S_2}$ is a **S-valued edge homomorphism** if

(i) $\alpha : V_1 \to V_2$ **is a graph homomorphism**
(ii) $\beta : S_1 \to S_2$**is a semiring homomorphism** with
$\beta(\psi_1(v_i, v_j) = \psi_2(\alpha(v_i), \alpha(v_j))\ \forall (v_i, v_j) \in E_1$

Example 3.5. Let S_1, S_2, $G_1^{S_1}$, $G_2^{S_2}$, α, β be as in example 3.2

Then α is a graph homomorphism and β is a semiring homomorphism.
Now $\beta(\psi_1(v_1,v_3)) = \beta(a) = f = \psi_2(\alpha(v_1), \alpha(v_3)) = \psi_2(u_5, u_4)$
$\beta(\psi_1(v_1, v_4)) = \beta(a) = f = \psi_2(\alpha(v_1), \alpha(v_4)) = \psi_2(u_5, u_6)$
$\beta(\psi_1(v_2, v_5)) = \beta(a) = f = \psi_2(\alpha(v_2), \alpha(v_5)) = \psi_2(u_2, u_3)$
$\beta(\psi_1(v_2, v_4)) = \beta(b) = h = \psi_2(\alpha(v_2), \alpha(v_4)) = \psi_2(u_2, u_6)$
$\beta(\psi_1(v_3, v_5)) = \beta(a) = f = \psi_2(\alpha(v_3), \alpha(v_5)) = \psi_2(u_4, u_3)$
Therefore $\beta(\psi_1(v_i, v_j)) = \psi_2(\alpha(v_i), \alpha(v_j))\ \forall\ (v_i, v_j) \in E_1$
$\Rightarrow\phi=(\alpha, \beta)$ is a S-valued edge homomorphism.

Remark 3.6. From the above example we observe that S-valued edge homomorphism need not be, in general, preserve S-edge regularity and hence S-regularity.

Definition 3.7.$\phi = (\alpha,\beta) : G_1^{S_1} \to G_2^{S_2}$ is an onto homomorphism if both α and β are onto homomorphism.

Remark 3.8$\phi=(\alpha, \beta) : G_1^{S_1} \to G_2^{S_2}$ is an onto S-valued vertex (edge) homomorphism if $G_2^{S_2}$ is a S-valued veretex (edge) homomorphic image of $G_1^{S_1}$.That is $G_2^{S_2} = \phi(G_1^{S_1})$.

Theorem 3.9. If $\beta : S_1 \to S_2$ be a semiring homomorphism where S_1 and S_2 are semirings and if $a \preccurlyeq b$ in S_1 then $\beta(a) \preccurlyeq \beta(b)$. That is β preserves canonical preorder.

Proof: Let $\beta : S_1 \to S_2$ be a semiring homomorphism and $a \preccurlyeq b$ in S_1
Then there exist $c \in S_1$ such that $a + c = b \in S_1$
Therefore $\beta(a + c) = \beta(b) \in S_2$
$\Rightarrow\beta(a) + \beta(c) = \beta(b) \in S_2$
$\Rightarrow\beta(a) \preccurlyeq \beta(b)$
That is β preserves canonical pre order.

Theorem 3.10. If a map $\phi = (\alpha,\beta) : G_1^{S_1} \to G_2^{S_2}$ is a S-valued vertex homomorphism then it is also a S-valued edge homomorphism.

Proof : Let $G_1^{S_1}$ = (V_1, E_1,σ_1, ψ_1) and $G_2^{S_2}$ = (V_2, E_2, σ_2, ψ_2) be two given S-valuedgraphs.Let $\phi = (\alpha, \beta)$: $G_1^{S_1} \to G_2^{S_2}$ be a S – valued vertex homomorphism.

Therefore $\alpha : V_1 \to V_2$ is a graph homomorphism and $\beta : S_1 \to S_2$ is a semiring homomorphism with $\beta(\sigma_1(v_1) = \sigma_2(\alpha(v_i))\ \forall\ v_i \in V$ .... (1).

Claim: $\beta(\psi_1(v_i, v_j)) = \psi_2(\alpha(v_i), \alpha(v_j))\ \forall\ (v_i, v_j) \in E_1$.
For, Let $(v_i, v_j) \in E_1$. Since α in a graph homomorphism, $(\alpha(v_i), \alpha(v_j)) \in E_2$.

In particular, $\beta(\sigma_1(v_i)) = \sigma_2(\alpha(v_i))$ and $\beta((\sigma_1(v_j)) = \sigma_2(\alpha(v_j))$. Without loss of generality, let us assume $\sigma_1(v_i) \preccurlyeq \sigma_1(v_j)$.By above theorem $\beta(\sigma_1(v_i)) \preccurlyeq \beta(\sigma_1(v_j))$ .... (2).

Therefore $\psi_1(v_i, v_j) = \min\{\sigma_1(v_i), \sigma_2(v_j)\} = \sigma_1(v_i)$

$\Rightarrow \beta(\psi_1(v_i, v_j)) = \beta(\sigma_1(v_i)) = \sigma_2(\alpha(v_i))$............(3).

Now, $\psi_2(\alpha(v_i)), \alpha(v_j)) = \min\{\sigma_2(\alpha(v_i)), \sigma_2(\alpha(v_j))\}$

$= \min\{\beta(\sigma_1(v_i)), \beta(\sigma_1(v_i))\} = \beta(\sigma_1(v_i)) = \sigma_2(\alpha(v_i))$ .... (4)

From (3) and (4), $\beta(\psi_1(v_i, u_j) = \psi_2(\alpha(v_i), \alpha(v_j))$.

Similarly if $\sigma_1(v_j) \preccurlyeq \sigma_1(v_i)$, then also $\beta(\psi_1(v_i, u_j)) = \psi_2(\alpha(v_i), \alpha(v_j))$.

Since $(v_i, v_j) \in E_1$ is arbitrary, $\beta(\psi_1(v_i, v_j)) = \psi_2(\alpha(v_i), \alpha(v_j)) \forall (v_i, v_j) \in E_1$.

$\Rightarrow \beta : S_1 \rightarrow S_2$ is a semiring homomorphism with

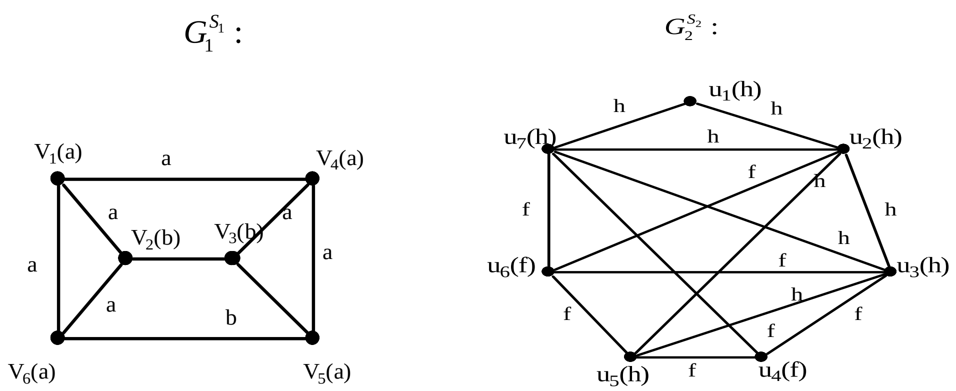
$\beta(\psi_1(v_i, v_j)) = \psi_2(\alpha(v_i), \alpha(v_j)) \forall (v_i, v_j) \in E_1$.

Therefore $\phi = (\alpha,\beta) : G_1^{S_1} \to G_2^{S_2}$ is a S – valued edge homomorphism.

Remark 3.11. Converse of the above theorem is not true.

Thatis, every S-valued edge homomorphism mapping is not a S-valued vertex homomorphism.

For, consider a semiringhomomorphism $\beta : S_1 \rightarrow S_2$as in example 3.2. Let $G_1^{S_1}$ and $G_2^{S_2}$ be as follows:



Define α : $V_1 \rightarrow V_2$ by $v_1 \mapsto u_3$; $v_2 \mapsto u_5$; $v_3 \mapsto u_2$; $v_4 \mapsto u_6$; $v_5 \mapsto u_7$; $v_6 \rightarrow u_4$. Clearly, $(v_1, v_4) \mapsto (u_3, u_6)$; $(v_1, v_2) \mapsto (u_3, u_5)$; $(v_1, v_6) \mapsto (u_3, u_4)$; $(v_2, v_3) \mapsto (u_5, u_2)$; $(v_2,v_6) \mapsto (u_5, u_4)$; $(v_3, v_4) \mapsto (u_2, u_6)$; $(v_3, v_5) \mapsto (u_2, u_7)$; $(u_4, u_5) \mapsto (u_6, u_7)$; $(v_5, v_6) \mapsto (u_7, u_4)$.

$\Rightarrow (v_i, v_j) \in E_1 \Rightarrow (\alpha(v_i), \alpha(v_j)) \in E_2 \forall (v_i, v_j) \in E_1$.

Therefore α is a graph homomorphism.

Now $\beta(\psi_1(v_1,v_4)) = \beta(a) = f = \psi_2(\alpha(v_1),\alpha(v_4)), = \psi_2(u_3,u_6)$

$\beta(\psi_1(v_1,v_2)) = \beta(a) = f = \psi_2(\alpha(v_1),\alpha(v_2)), = \psi_2(u_3,u_5)$

$\beta(\psi_1(v_1,v_6)) = \beta(a) = f = \psi_2(\alpha(v_1),\alpha(v_6)), = \psi_2(u_3,u_4)$

$\beta(\psi_1(v_2,v_3)) = \beta(b) = h = \psi_2(\alpha(v_2),\alpha(v_3)), = \psi_2(u_5,u_2)$

$\beta(\psi_1(v_2,v_6)) = \beta(a) = f = \psi_2(\alpha(v_2),\alpha(v_6)), = \psi_2(u_5,u_4)$

$\beta(\psi_1(v_3,v_4)) = \beta(a) = f = \psi_2(\alpha(v_3),\alpha(v_4)), = \psi_2(u_2,u_6)$

$\beta(\psi_1(v_3,v_5)) = \beta(b) = h = \psi_2(\alpha(v_3),\alpha(v_5)), = \psi_2(u_2,u_7)$

$\beta(\psi_1(v_4,v_5)) = \beta(a) = f = \psi_2(\alpha(v_4),\alpha(v_5)), = \psi_2(u_6,u_7)$

$\beta(\psi_1(v_5,v_6)) = \beta(a) = f = \psi_2(\alpha(v_5),\alpha(v_6)), = \psi_2(u_7,u_4)$

$\Rightarrow \beta(\psi_1(v_i, v_j)) = \psi_2(\alpha(v_i), \alpha(v_j)) \forall (v_i,v_j) \in E_1$ ...... (*).

$\Rightarrow\beta$is a semiring homomorphism satisfying equation (*).

Therefore $\phi = (\alpha, \beta) : G_1^{S_1} \rightarrow G_2^{S_2}$ is a S – valued edge homomorphism.

Now, In Particular, $\sigma_1(v_1) = a \Rightarrow \beta(\sigma_1(v_1)) = \beta(a) = f$ and $\sigma_2(\alpha(v_1)) = \sigma_2(u_3) = h$.

Therefore $\beta(\sigma_1(v_1)) \neq \sigma_2(\alpha(v_1)) \Rightarrow \phi = (\alpha,\beta)$ is not a S-valued vertex homomorphism.

It is a S – valued edge homomorphism but not a S – valued vertex homomorphism.

Remark 3.12. In the above example, if we assign $\sigma_2(u_5) = h$,

Then$\beta(\sigma_1(v_1)) \neq \sigma_2(\alpha(v_1))$ .... (1).

And $\beta(\psi_1(u_1, u_2)) = \beta(a) = f$; $\psi_2(\alpha(v_1), \alpha(v_2)) = \psi_2(u_3, u_5) = h$.

Therefore $\beta(\psi_1(u_1, u_2)) \neq \psi_2(\alpha(v_1), \alpha(v_2))$ .... (2)

From (1) and (2), $\phi = (\alpha,\beta)$ is neither a S – valued vertex nor edge homomorphism.

Remark 3.13. If α is not a graph homomorphism then we can't talk about S-valued vertex or edge homomorphism.

Definition 3.14. A mapping $\phi = (\alpha,\beta)$: $G_1^{S_1} \rightarrow G_2^{S_1}$ is said to be a S-valued semi homomorphism if it is both S –valued vertex and edge homomorphism.

Example 3.15. Consider $\phi = (\alpha, \beta) : G_1^{S_1} \rightarrow G_2^{S_1}$ in example 3.2.

From examples 3.2 and 3.5, ϕ is both S-valued vertex and edge homomorphism.

Therefore it is a S-valued semi homomorphism.

Theorem 3.16. A mapping $\phi = (\alpha,\beta) : G_1^{S_1} \rightarrow G_2^{S_1}$ is a S-valued semi homomorphism if and only if it is a S – valued vertex homomorphism.

Proof: Let ϕ be a S – valued vertex homomorphism.

By theorem 18, it is a S-valued edge homomorphism.

Therefore ϕ is a S –valued semi homomorphism.

By the definition of S –Valued semi homomorphism, converse follows.

Example 3.17. Consider the semiring (S = {0, a, b, c}, +, .) in which '+' and '.' are defined as follows:

+	0	a	b	c
0	0	a	b	c
a	a	b	c	c
b	b	b	c	c
c	c	c	c	c

.	0	a	b	c
0	0	0	0	0
a	0	a	b	c
b	0	b	c	c
c	0	c	c	c

Here 0 is an additive identity element. a is a multiplicative identity element.

That is ais a unit element.

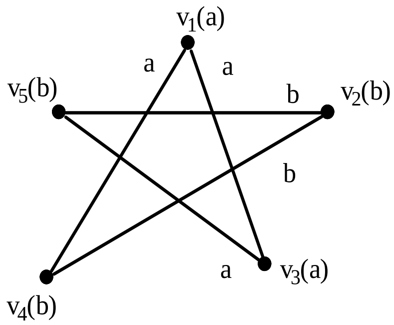
Consider the identity map $\beta : S \rightarrow S$ which maps $0 \mapsto 0$; $a \mapsto a$; $b \mapsto b$; $c \mapsto c$.

Then zero element and unit element are mapped into zero and unit element. And $\beta(a + b) = \beta(a) + \beta(b)$; $\beta(a.b) = \beta(a). \beta(b)$ $\forall a, b \in S$.

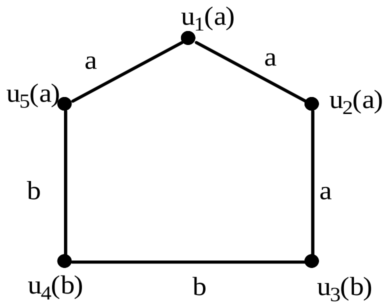
$\Rightarrow\beta$is a semiring homomorphism which is both one-one and onto.

Let $G_1^{S_1}$ and $G_2^{S_2}$ be as follows:

G_1^{S} :



$G_2^{S_2}$:



Define $\alpha : G_1^{S_1} \to G_2^{S_1}$ as

$v_1 \mapsto u_1 : v_2 \mapsto u_4; v_3 \mapsto u_2 ; v_4 \mapsto u_5;\ v_5 \mapsto u_3$. then

$(v_i, v_j) \in E_1 \Rightarrow (\alpha(v_i), \alpha(v_j)) \in E_2$.

$\Rightarrow$a is a graph homomorphism.

Clearly α is one-one and onto.

Therefore a is a graph isomophism and β satisfies, $\beta(\sigma_1(v_i)) = \sigma_2(\alpha(v_i)) \forall v_i \in V_1$.

$\beta(\psi_1(v_i,v_j)) = \psi_2(\alpha(v_i), \alpha(v_j)) \forall (v_i, v_j) \in G$.

Therefore $\phi = (\alpha, \beta)$ is as S-valued semi homomorphism.

Remark 3.18. A S-valued semi homomorphism $\phi = (\alpha, \beta)$ is a S-valued homomorphism if α is a graph isomorphism.

Example 3.19. Consider the above example in which α is an isomorphism, Therefore $\phi = (\alpha, \beta)$ which is defined in the above example is a S-valued homomorphism.

Theorem 3.20. If $\phi = (\alpha, \beta)$ is a S – valued vertex homomorphism from a vertex regular graph $G_1^{S_1}$ with S_1 – vertex set {a} into a S – valued graph $G_2^{S_2}$ then ϕ ($G_1^{S_1}$) is a S-vertex regular graph with S-vertex set {β(a)}.

Proof: Let $\phi = (\alpha, \beta) : G_1^{S_1} \to G_2^{S_2}$ be a S-valued vertex homomorphism

Where $G_1^{S_1} = (V_1, E_1, \sigma_1, \psi_1)$, $G_2^{S_2} = (V_2, E_2, \sigma_2, \psi_2)$.

Then α is a graph homomorphism preserving edges and β is a semiring homomorphism with $\beta(\sigma_1(v_i)) = \sigma_2(\alpha(v_i)) \ \forall\ v_i \in V_1$.

Since $G_1^{S_1}$ is a S – valued vertex regular graph, $\sigma_1(v_i) = a$, for some $a \in S$ and for all $v_i \in V_1$. Therefore $\beta(\sigma_1(v_i)) = \beta(a)\ \forall\ i \Rightarrow \sigma_2(\alpha(v_i)) = \beta(a) \forall$ i.

Let $u_i \in \phi(G_1^{S_1})$ be arbitrary. Thenthere exist $v_i \in V_1$ such that $u_i = \alpha(v_i)$. (sinceα is graph homomorphism)

Since β is a semiringhomomorphism ,$\beta(\sigma_1(v_i)) = \sigma_2(\alpha(v_i))$.

$\Rightarrow \beta(a) = \sigma_2(u_i)$.Since u_i is arbitrary, $\sigma_2(u_i) = \beta(a)\ \forall u_i \in \phi(G_1^{S_1})$

Therefore $\phi(G_1^{S_1})$ is a S-valued vertex regular graph with S-vertex set {β(a)}.

4. CONCLUSION

All the result proved for S-valued homomorphisms coincide with S- valued vertex and edged homomorphisms. Further study is required to count the number of S –valued morphisms.

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