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Free Vibration Analysis of Stiffened and Unstiffened Membranes

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Abstract

This paper presents the vibration studies of flat thin membranes. Membrane structures are widely used nowadays in various fields like aerospace, medicine, musical instruments due to their various desirable properties. Hence it is important to study their vibration behavior. A free vibration analysis of flat pre-stressed membranes is carried out using finite element analysis code ANSYS and the results are compared with analytical solutions. Good agreement between the two solutions is obtained. Furthermore, vibration analysis of both stiffened and unstiffened membranes is carried out, the results are compared and necessary conclusions are drawn.

Nomenclature:

FEA : Finite element analysis.

t : Time in seconds.

w(x,y,t) : Field variable.

T : Tension for pre-stressing in N/m.

H : thickness the membrane in mm.

w(x,y) : Eigen function.

W : Circular eigen frequency in Hz.

Keywords: *Membrane, Prestress, Stiffness, Frequency, Modes, Vibrations.*

Introduction

A membrane is essentially a thin shell with no flexural stiffness. Consequently a membrane cannot resist any compression at all. However, membrane theory accounts for tension and compression stresses. In membrane theory only the in-plane stress resultants are taken into account. This study presents the modal analysis for predicting the behaviour of various shaped thin membranes of various materials which are optimally subjected to pre-stress to render them to behave as structural members rather than bending or moments.

Membrane materials

Table1: Properties of membrane materials (Ruggiereo etal 2003, Srivastava et al 2008).

Sl no	Name of the Material	Mass Density [kg/m^3]	Young's Modulus [N/m^2]	Poisso-n's ratio
1	Kevlar	790	$11.9 x 10^9$	0.3
2	Kapton	1420	$2.5 x 10^9$	0.34
3	Mylar	1390	$8.81 x 0^9$	0.38
4	PVDF	1780	$2.8 x 10^9$	0.32

Dynamics of Membrane

A membrane has no compression or bending stiffness, therefore it has to be pre-stressed to act as a structural element. For analysis of membrane structure following assumptions are made

1. Effect of gravity on the membrane is negligible.
2. Displacement is only in vertical direction.
3. Membrane is thin enough to neglect its volume and only consider its area.
4. Magnitude of pre-stress remains constant and mass density assumed is uniform throughout the membrane.

For simplicity, consider a membrane stretched between x[0,a] and y[0,b]. If the membrane is fixed on all the edges, a solution of the unforced dynamics of the membrane is formed as,

$$w(x,y,t) = W(x,y)e^{i\dot{\omega}t}$$

where W (x,y) is an eigen function and $\dot{\omega}$ is the corresponding eigen frequency. Since the actual solution can be found from the real part of the equation, by substitution we get,

$$w_{xx} + w_{yy} + \frac{\omega^2}{c^2}w = 0, \qquad \eta^2 w + \frac{\omega^2}{c^2}w = 0 \qquad \text{where, } \eta^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \qquad \text{and} \quad c = \sqrt{\frac{T}{\rho h}}$$

The four boundary conditions are applied as,

W(0,y) = 0; W(a,y)=0; W(x,0) = 0; and W(x,b)=0.

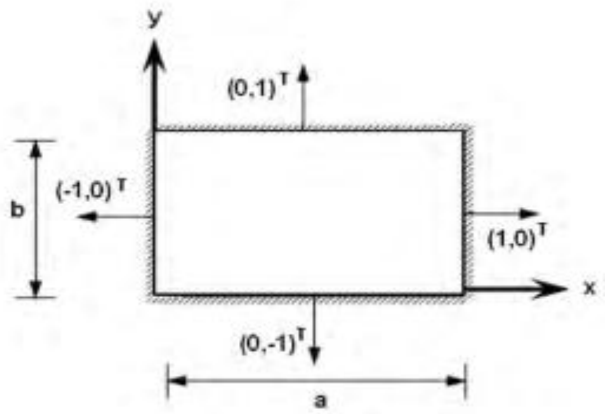


Figure 2: Rectangular membranes with boundary normals in Cartesian coordinates.

The general equation can be obtained as,

$$W(x,y) = (A_1 \cos\alpha x \cos\beta y) + (A_1 \sin\alpha x \sin\beta y) + (A_1 \sin\alpha x \cos\beta y) + (A_1 \sin\alpha x \sin\beta y)$$

Using boundary conditions and simplifying we get,

$$W(x,y) = (A_4 \sin\alpha x \sin\beta y)$$

$\alpha = \frac{m\pi}{a}$ and $\beta = \frac{n\pi}{b}$ where m, n$\in$ [1,∞].

The eigen functions are obtained from the above equations as $W^{(m,n)} = \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}$ where m, n ϵ $[1, \infty)$. The circular frequency equation of the (m,n) under the condition of $\alpha^2 + \beta^2 = \frac{\omega^2}{c^2}$, give $\omega_{(m,n)} = \pi c\sqrt{\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)}$.

Results and Discussion

FEA For Free Vibration Analysis of an Un-stiffened Square Membrane

Dimensions and Parameters

Figure 3: The Meshed Square membrane

Length =0.2m

Width =0.2m

Material : Kevlar

Pre-stress Applied : 10 N/m

No. of Elements : 1600 (obtained after Convergence studies)

Boundary Conditions : The membrane is fixed on all the edges.

Table: 2 FEA and analytical results

Mode No.	FEA in Ansys (Hz)	Analytical Method (Hz)
1	39.82	39.78
2	62.88	62.89
3	63.11	63.18
4	79.86	79.83
5	89.65	89.69
6	89.75	89.84

Mode Shapes

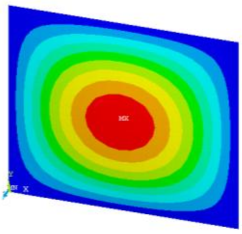


Figure 4: (1,1) mode 39.82 Hz

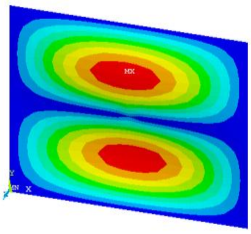


Figure 5: (1,2) mode 62.88 Hz

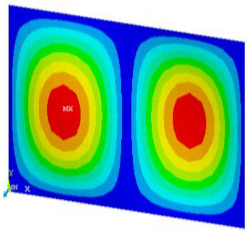


Figure 6: (2,1) mode 63.11 Hz

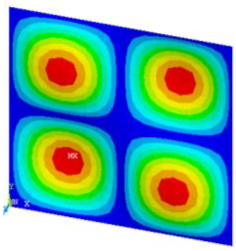


Figure 7: (2,2) mode 79.86Hz

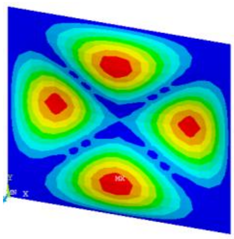


Figure 8: (1,3) mode 89.65 Hz

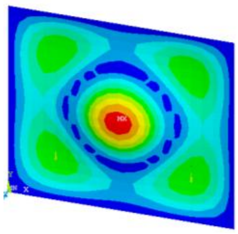


Figure 9: (3,1) mode 89.84 Hz

Free Vibration Analysis of Stiffened Membrane With Steel Stiffeners and Composite (PVDF) Stiffeners.

Dimensions and Parameters

Length =0.2m

Width =0.2m

Material : Kevlar

Pre-stress Applied : 10 N/m

No. of Elements : 1600

Stiffeners used : Steel and PVDF.

Boundary Conditions : The membrane is fixed on all the edges.

While modelling this in Ansys, we attach the square section(10mm x 10mm) steel beam to the existing membrane and to the same nodes.

Table: 3 Natural frequencies of membrane with steel stiffeners and composite (PVDF) stiffener.

Mode No.	Natural Frequency (Hz) steel stiffener	Natural Frequency (Hz) Composite stiffener
1	25.998	34.210
2	35.459	38.124
3	45.073	52.173
4	47.098	53.846
5	51.390	58.361
6	59.887	68.916

Mode Shapes

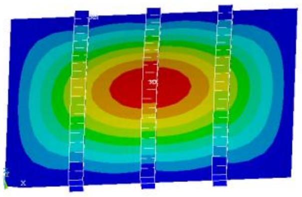


Figure 10: (1,1) mode

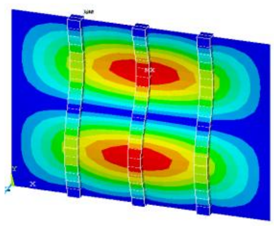


Figure 11: (1,2) mode

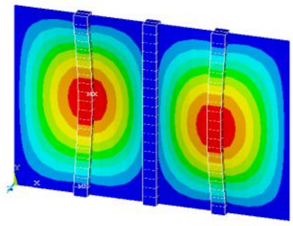


Figure 12: (2,1) mode

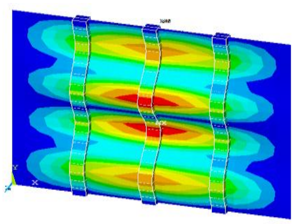


Figure 13: (2,2) mode

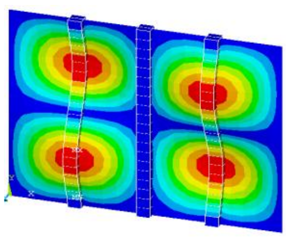


Figure 14: (1,3) mode

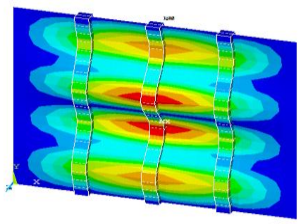


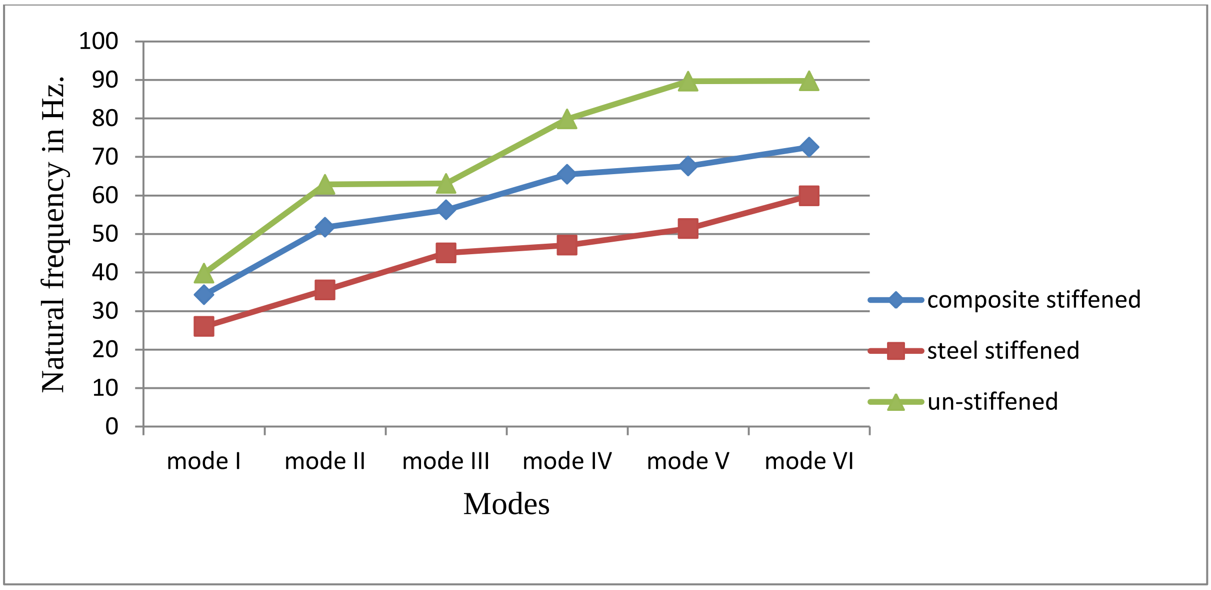
Figure 15: (3,1) mode

Comparison between stiffened and un-stiffened membranes

When we compare the natural frequencies obtained by applying the steel and composite stiffeners and un-stiffened membranes, we can make the following observations:

a) The natural frequencies of steel stiffened membranes are pretty low compared to that of un-stiffened membranes. It is due to the fact that instead of stiffness, the mass of steel is the one which makes its presence felt because of high density. And hence the frequency significantly decreases.
b) When we apply the composite (PVDF) stiffener, the frequencies are
c) higher than that compared to the steel stiffened membrane but still lower than the un-stiffened membrane because the density of PVDF is lower than the steel but its young's modulus is lower than the un-stiffened membrane material (Kevlar) .
d) Hence, it can be concluded that by using the composite stiffener which has higher young's modulus and much lower density, the natural frequency can be further increased.

Figure 16: Comparison of natural frequencies of stiffened and un-stiffened membranes.



Conclusions and Scope for future work

The modeling of free vibration analysis of membranes of different materials was performed using ANSYS FEA package. It was found that the results obtained were showing the good match with the analytically calculated values. Also, nowadays, both stiffened and un-stiffened membranes are found
to have various applications, the studies were made to compare stiffened and un-stiffened membranes and decisive conclusions on the effect on natural frequencies were made. The present work demonstrates the free vibration analysis of membranes using ANSYS FEA package. In future, the harmonic analysis (forced vibration analysis) can be carried out. More studies can be carried out based on the specific applications. More powerful FEA tools like ABAQUS can be used for more dynamic visualisation of modes. Based on the present work the membrane structures can be designed for optimal vibrations. Also, studies can be taken up for varying temperature conditions like cryogenic temperatures and very high temperatures.

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