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Implementation of Factorized Kalman Filter for Target Image Tracking

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ABSTRACT
Filtering is useful for many engineering and aerospace applications like satellite orbit estimation or aircraft state estimation. Factorized Kalman filtering strategy which was originally developed for using it in spacecraft navigation is discussed in this paper. This algorithm, involves the two aspects of automatic target recognition which are location and identity estimation of a moving target in the generated images and centroid tracking for target state estimation using sensor data. For tracking the centroid of the grey level image factorized Kalman filter is used. A MATLAB code is developed and the simulated data results indicate a good tracking performance for images. It was observed that filter generates robust tracks and its results are validated using true data.
Keywords- *Identity estimation, Centroid tracking, Automatic target recognition (ATR), Percentage fit error, Kalman filter(KF), Centroid tracking UD filter (CTUDF).*

1. INTRODUCTION

One of the key modules of present and future defence weapon systems is Automatic target recognition (ATR) [4]. It is used in independent as well as manned automobile missions. ATR is very much needed for the safety and early cautionary of the professed threat in air traffic control. One of the essential components of ATR process involves programmed target acquisition, identification and tracking by continuously processing a sequence of images. In these image processing applications, suitable method of registration, detection, sorting, and feature estimation would be required for target position and identity assessment. Further, if the targets are moving, algorithms for tracking targets would contain features like segmentation, centroid computation and tracking by means of appropriate algorithms. The basic level of tracking actions involves numeric measures having linear and nonlinear estimation methods, pattern recognition methods. Different algorithms like kalman filter, extended kalman filter are used to give relevant information about position and velocity of the target. Different data could be combined in an appropriate way, to make proper inference about the target details. This will increase the assurance in the ATR method.

This paper presents two parts involved in ATR, one is identity and position assessment of an target using sensor data, and second is tracking the centroid ofthe target state for estimation using sensor statistics. This aspect is explained using a centroid estimation target tracking algorithm with factorized Kalman filter.

Pursuing the moving objects as image data includes processing pictures starting from a target of interest and creating at each time step, an estimate of the target's existing position as well as velocity trajectories. Uncertainties happening in detection of target position and movementare modelled by adding Gaussian noise with a known standard deviation.

2. TARGET IMAGE TRACKING

The traditional Kalman filter result fluctuates due to one or more of the following reasons:

I. Modelling faults (because of nonlinear method)
II. Incorrect a priori data
III. Finite word length implementation of the filter.

For handling (i) a properly tuned extended Kalman filter should be used. If feasible, for Kalman filter accurate mathematical models of the system should be used. For handling (ii) proper tuning should be done. Adaptive tuning methods should be used. For (iii) factorization filtering methods is used to substantiate the fluctuationsdue to finite word length.

In the Kalman filter, equation for covariance (a posteriori) is especially ill-conditioned. Due to round off errors in computation and their propagation, the covariance matrix (P) could be rendered non-positive definite, whereas theoretically it should be at least semi-positive definite. In addition, P should be symmetric, but during computation it could lose this property. All these will lead the Kalman filter to diverge and the filter estimate will not converge in the sense of mean square to the true state. These effects are circumvented by implementing a Kalman filter in its factorized form. These algorithms do not process P in its original form, but process its square root. Such factorization indirectly preserves the symmetry and ensures the non-negativity of P.

2.1 Centroid Tracking Factorized Kalman Filter

Factorized Kalman filter is also referred to as UD filter. Centroid tracking UD filter (CTUDF) is a process to yield the utmost exact and ample data about an entity, action or event. As an expert technology, UD factorization [3] is the incorporation and application of numerous traditional disciplines and fresh zones of engineering with communication and assessment theory, uncertainty controlling, estimation philosophy, digital signal processing, computer knowledge and artificial cleverness. CTUDF results shows that it has strong operational performance, prolonged spatial resolution, condensed uncertainty, consistency and dimensionality superior than Kalmanfiter [6]. U-D factorized Kalman filter philosophy is defined. The scheme is named CTUDF strategy which is verified with true data.

2.2 CTUDF Filtering Algorithm

In CTUDF U and Dare the factors of the covariance matrix P of the Kalman filter.U is a unit upper triangular matrix and D is a diagonal matrix.

The problem addressed in this paper is represented by the following set of equations. [8]

State model:

$$X(i+1) = \emptyset X(i) + Gw(i) \qquad (1)$$

Measurement model:

$$Z(i) = HX(i) + v(i) \qquad (2)$$

Here, X is the state vector, w is the process noise with zero mean and covariance matrix Q, Z is the measurement vector and v is the measurement noise with zero mean and covariance matrix R, all of appropriate dimensions.

The state transition matrix of the state model (1) is given by

$$\emptyset = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (3)$$

Where T=1 second (sampling interval). The state model H is given by (4)

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad (4)$$

State vector X has position and velocity as its components. This filter is implemented in the factorized form for the present application. The tracking problem is related to estimation of the position and velocity of a moving target (based on the available measurements) using CTUDF filter.

This algorithm consists of two parts:

- Time Propagation and
- Measurement Update algorithms [7]

a) Time Propagation Algorithm

Time propagation algorithm directly and efficiently produces the required U,D factors, including the previous effect of U,D factors and process noise. It preserves the symmetry of P matrix.

Prediction of state vector estimate is given by (5)

$$\tilde{x}(i+1) = \emptyset\hat{x}(i) \quad (5)$$

Covariance update equation is given by (6)

$$\tilde{P}(i+1) = \emptyset\hat{P}(i)\emptyset^T + GQG^T \quad (6)$$

With $\hat{P} = \hat{U}\hat{D}\hat{U}^T$, the times update factors $\tilde{U}$ and $\tilde{D}$ are obtained through modified Gram-Schmidt orthogonalization process. The matrix U is an upper triangular matrix with unit elements on its main diagonal and D is a diagonal matrix. Covariance and gain processing algorithms, operating on U and D factors of state error covariance matrix P, are a technique for implementing "square root filtering" without requiring computation of square roots. The U-D Kalman filtering algorithm is considered efficient, stable and accurate for real-time applications.

The vector W is computed using [7]

$$W = [\emptyset\hat{U}|G_A], \quad \bar{D} = \mathrm{diag} \quad (7)$$

With$W^T = [w1, w2, \dots, wn]$

The U,D factors of $\tilde{P} = \widetilde{W}\tilde{D}\widetilde{W}^T$ is computed.

For$j = n, n-1, \dots, 2$, the following equations are recursively evaluated as shown below.

$$\tilde{D}_j = < w_j^{(n-j)}, w_j^{(n-j)} >_D$$

$$\tilde{U}(k,j) = \left(\frac{1}{\tilde{D}_j}\right) < w_k^{(n-j)}, w_j^{(n-j)} >_D$$

$$w_k^{(n-j+1)} = w_k^{(n-j)} - \tilde{U}(k,j)w_j^{(n-j)}$$

$$\tilde{D}_1 = < w_1^{(n-1)}, w_1^{(n-1)} >_D \quad (8)$$

Where $k = 1, \dots, (j-1)$

Here subscript D qualifies the weighted inner product with respect to D.

The measurement update part continues the time propagation.

b) Measurement Update Algorithm:

The measurement update in Kalman filtering combines a priori estimate $\tilde{x}$ and error covariance $\tilde{P}$ with scalar observation $z = a^T x + v; \quad a^T = H;$

To construct an updated (filtered state) estimate and covariance following equations are used:

$$K = \frac{\tilde{P}a}{\alpha}$$

$$\hat{x} = \tilde{x} + K(z - a^T\tilde{x})$$

$$\alpha = a^T\tilde{P}a + r$$

$$\hat{P} = \tilde{P} - Ka\tilde{P} (9)$$

Where,$\tilde{P} = \tilde{U}\tilde{D}\tilde{U}^T$, ‘a’ is the measurement matrix, ‘r’ is the measurement noise covariance (for scalar data processing) and ‘z’ is the string of noisy measurements. The gain K and updated covariance factors $\hat{U}$ and $\hat{D}$ can be obtained from the following equations (10).

$$f = \tilde{U}^T a, \qquad f^T = (f1, \dots, fn);$$

$$v = \tilde{D}f,$$

$$v_k = \tilde{d}_k f_k k = 1, 2, \dots, n$$

$$\hat{d}_1 = \tilde{d}_1 r/\alpha_1,$$

$$\alpha_1 = r + v_1 f_1, \quad K_2^T = (v_1 0 \dots 0) \quad (10)$$

For $j = 2, \dots, n$, recursively the following equations (11) are evaluated:

$$\alpha_j = \alpha_{j-1} + v_j f_j,$$

$$\hat{d}_j = \frac{\tilde{d}_j \alpha_{j-1}}{\alpha_j} \hat{u}_j = \tilde{u}_j + \gamma_j K_j$$

$$\gamma_j = -f_j/\alpha_{j-1}, K_{j+1} = K_j + v_j \tilde{u}_j \ (11)$$

Where $\tilde{U} = [\tilde{u}_1, \dots, \tilde{u}_n], \hat{U} = [\hat{u}_1, \dots, \hat{u}_n]$, and Kalman gain is given by$K = K_{n+1}/\alpha_n$.

Here $\tilde{d}$ is predicted diagonal element, and $\hat{d}_j$ is the updated diagonal element of the D matrix. The U-D filter described above is developed in MATLAB. It has been validated using true data. [1]

2.3 Tracking and Estimation

a. Target Image Generation

The mathematical basis for generation of target image synthetically is described below:

Two-dimensional array of pixels are considered

$$m = m_\varepsilon + m_\delta \quad (12)$$

Eachpixel is represented by a single index i = 1... m and the intensity ‘I’ of pixel ‘i’ is given by

$$I_i = s_i + n_i \quad (13)$$

Where, s_i is the target intensity and n_i is the noise intensity in pixel i, which is assumed to be Gaussian with zero mean and covariance σ^2 [5]. The total target-related intensity is given by:

$s = \sum_{i=1}^{m} s_i$ (14)

If the number of pixels covered by the target is denoted by m_s, then the average target intensity over its extent is given by:

$\mu_s = s/m_s$ (15)

The average pixel signal to noise ratio (over the extent of the target) is

$r = \mu_s/\sigma$ (16)

Using Equations (12) - (16), the synthetic images are generated by using the following inputs:

- Target pixel intensity (s_i): $N(\mu_t, \sigma_t^2)$
- Noise pixel intensity (s_i): $N(\mu_n, \sigma_n^2)$
- Target: Rectangle (base NX and height NY)
- Position of the target in each scan: (x-position and y-position)

b. *Segmentation and Centroid detection of the target*

Particle segmentation is used to separate the target (object of interest) from background. It is assumed that the pixel intensities are discretised into 256 grey levels. [2] Particle segmentation is done in two steps: (i) The grey level image is converted into binary image using lower and upper threshold limits of the target. These thresholds of target are determined using the pixel intensity histograms from the target and its surroundings, and (ii) The detected pixels are grouped into clusters with nearest neighbour technique. The target image is converted into binary image. The MATLAB inbuilt function regionprops() is used to find the centroid of the target.

c. *Target Image Centroid Tracking Algorithm*

In general gating and data association enable tracking in multi target scenario. Target image centroid tracking in the presence of clutter is achieved using the nearest neighbour UD filter (NNUDF). Gating helps in deciding if an observation (which includes clutter, false alarms and electronic counter measures) is a probable candidate for track maintenance or track update. Data association is the step to associate the measurements to the targets with certainty when several targets are in the same neighbourhood.

The NNUDF is necessary for the centroid tracking application because in the neighbourhood of the predicted location for the target centroid during tracking, several centroids could be found due to splitting of the target cluster or due to noise clusters. The major advantage from factorized Kalman filter comes from the fact that the square root type algorithms process square roots of the covariance matrices and hence, they essentially use half the word length normally required by the conventional Kalman filters. In this filter, the covariance update formulae of the conventional basic kalman filter and the estimation recursion are reformulated, so that the covariance matrix does not appear explicitly. Specifically, we use recursions for *U* and *D* factors of covariance matrix $P = UDU^T$.

Computing and updating with triangular matrices involve fewer arithmetic operations and thus greatly reduce the problem of round off errors, which might cause ill-conditioning and subsequent divergence of the algorithm, especially if the filter is implemented on a finite word length machine.

3. PERFORMANCE EVALUATION OF CTUDF

The performance of CTUDF is evaluated in terms of the following position and velocity errors as (17) and (18):

(i) The Percentage Fit Error (PFE) (x or y position

$$PFE_x = 100 * \frac{norm(x-\hat{x})}{norm(x)}$$

$$PFE_y = 100 * \frac{norm(y-\hat{y})}{norm(y)} \quad (17)$$

(ii) The root sum square position error

$$RMSPE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\frac{(x_i-\hat{x}_i)^2+(y_i-\hat{y}_i)^2}{2}} \quad (18)$$

Similar formulae are applicable for velocity error (RMSVE).

The state errors with $\pm 2\sqrt{P_{i,i}}$ bounds, where P is the state error covariance matrix, where x and y are the measurements, $\hat{x}$ and $\hat{y}$ are the estimated target locations in x and y coordinates, respectively.

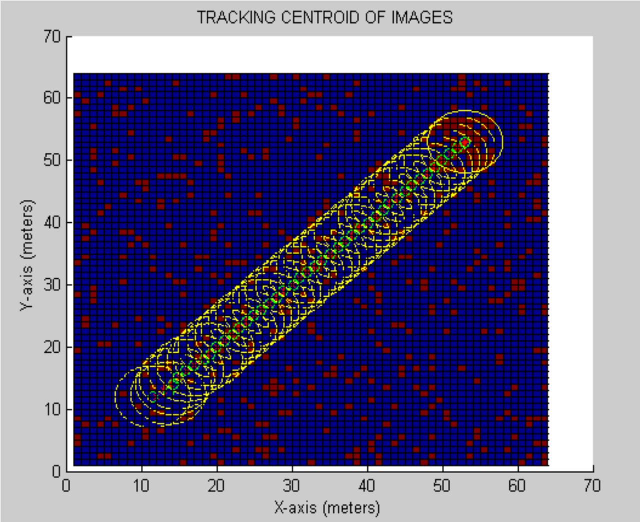

Figure 1: Tracking of centroid

A 2-D of 64 x 64 pixels is considered for the image. A 2-D array of pixels, which is modelled as white Gaussian random field with a mean μ and variance σ^2 is used to generate a rectangular target of size (9 x 9). The image is converted intobinary image using the upper (IU =110) and lower (IL=90) limits of a target layer, and then grouped into clusters by the nearest neighbour techniqueusing the optimal proximity distance (DP=2).

The total number of scans is 50. The image frame rate is one frame/s. The target and noise parameters used in this simulation are as follows: Target pixel intensity = $N(100,10^2)$, and Noise pixel intensity = $N(50,50^2)$. The centroid of each cluster is calculated and used for state estimation in the measurement update part of the CTUDFKF filters to track the target in clutter.

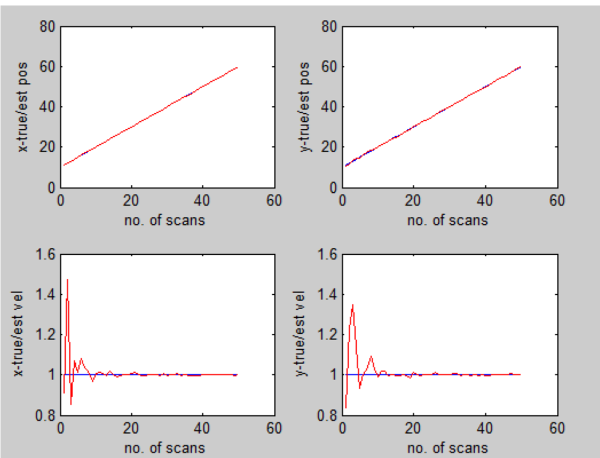

Figure 2: Plots of true and estimated values

The CTUDF algorithm includes track initiation and track deletion features which are essential in target tracking even in the presence of clutter. Figure shows the frame, which includes the estimated and true data of two targets in clutter. The frame shows the background clutter and the two synthetic target images at 50th scan.

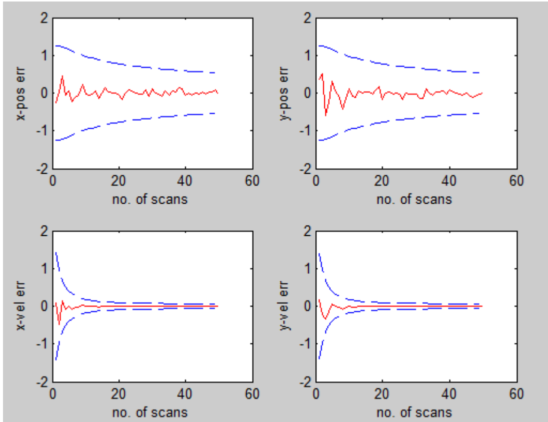

Figure 3: Plots of position amd velocity error

The plot results obtained using Matlab shows that obtained parameters are within the bounds and they tend to increase within creasing target noise as shown in the table1.

Table 1: Performance of CTUDF for different target noise variance (Vn)

Parameter	Vn=1	Vn=2	Vn=3	Vn=4	Vn=5
PFE_x	0.2860	0.3645	0.3908	0.4401	0.4873
PFE_y	0.4294	0.5128	0.5139	0.6389	0.6037
RMSPE	0.1977	0.2411	0.2474	0.2973	0.2973
RMSVE	0.1003	0.1157	0.1197	0.1527	0.1461

4. CONCLUSION

The factorized Kalman filter for tracking target image of interest captured from the sensor is presented. It is a square root type algorithm. So, the algorithm is most efficiently and simply mechanised by processing vector measurements (observables), one component at a time. CTUDF filter has proved its numerical reliability, accuracy and stability under different noise perturbations. The target position and velocity errors obtained are within bounds. This is more relevant for real-time implementation of target tracking using computers such as ATR and embedded applications.

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REFERENCES

1. Girija G, Raol JR, *"Singh J. Modelling and parameter estimation of dynamic systems",* London, The Institution of Engineering and Technology; 2004.
2. P. S. Naidu, Girija G. and J. R. Raol, *"Evaluation of data association and fusion algorithms for tracking in the presence of measurement loss",* AIAA Conference on Navigation, Guidance and Control, Austin, USA, 11-14 August 2003.
3. Girija G, Raol J R, Appavu Raj R, Banerjee P *"Intelligent multi sensor data fusion. Presented at the National Seminar on Intelligent and Autonomous Systems",* Computer Society of India, Mumbai, 1999.
4. BirBhanu, *"Automatic Target Recognition: State of the Art Survey"*, IEEE transactions on aerospace and electronics systems, vol. AES-22, No.4 july 1996.
5. Saha R K, *"Effect of common process noise on two-track fusion",*Journal of *Guidance, Control* and *Dynamics*, 19:829-835, *1996.*
6. Hall D L, *"Mathematical techniques in multi-sensor data fusion",* Boston, Artech House, pp.20-59, 1992.
7. Gerald J. Bierman, *"Factorization Methods for Discrete Sequential Estimation",* Jet Propulsion Laboratory, California Institute of Technology, Academic Press, New York, 1977.
8. Catherine I, Thornton, *"Triangular covariance factorizations for Kalman filtering",*Ph.D. Thesis, University of California at Los Angeles, School of Engineering, 1976.